THE DESIGN OF TUNABLE ACOUSTIC METAMATERIALS USING SCATTERING THEORY METHODS

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Abstract. A two-step method for designing acoustic metamaterials and calculating wave fields inside them is proposed. In the first step, the scattering coefficients are calculated, and in the second step, the specific design of the metamaterial elements is determined. The results of modeling a cloaking insulating shell and a lens with a tunable focus are presented.

Keywords: *tunable metamaterials, scattering theory, cloaking shell, Lippmann-Schwinger equation*

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INTRODUCTION

Acoustic metamaterials are artificially created media consisting of periodically or chaotically arranged elements with a characteristic size much smaller than the wavelength. Since the design of each such element can be specifically chosen, this opens up the possibility of designing media with many unusual properties not normally found in "conventional" solid materials. These include, for example, negative refractive index media [1, 2], auxetics [3], hiding structures [4 -7]. Metamaterials have found their application in many problems related to wave field control and the creation of absorbing coatings [8, 9].

It is of interest to introduce active elements [9] into the metamaterial structure. In one case, they can be a combination of sensors and acoustic field sources [10] connected by electronic circuits. This further extends the allowable range of wave properties of the medium as wave absorption limitations are removed. In addition, electronic circuits can perform signal processing, including nonlinear processing. In another case, active elements are used not to create an acoustic field, but to dynamically change the structure of the metamaterial, and thus its wave properties [11].

The main questions that arise in the study of metamaterials concern, on the one hand, the modeling of wave processes and the search for new interesting effects in these media, and, on the other hand, the principles of their creation in practice. To solve both problems, it is possible to model wave processes in the medium as a whole by breaking it into sections of sufficiently small size (e.g., by the finite element method, FEM). However, as a rule, the size of a metamaterial element should be significantly smaller than the effective wavelength in the metamaterial. On the other hand, such an element is often a structure whose parts are even smaller. This leads to the fact that the step of the chosen discretization grid turns out to be on average an order of magnitude smaller than in problems related to the modeling of continuous media, and the volume of calculations in such calculations increases dramatically.

Another solution is the introduction of effective parameters of the metamaterial (density, compressibility, sound velocity), which allows us to consider it as a continuous medium. Many effects found in metamaterials were originally considered using this approach [1,12]. To determine the spatial distribution of effective parameters, the method of transformational acoustics [13], which is actively used in the design of various hiding configurations, can be used. The difficulty of this approach is to find a practical design of the medium with given effective parameters. First of all, it is difficult to give a correct definition of such parameters for a discrete medium, especially if only a small number of its elements are arranged on the wavelength or they are located unevenly. Often, for this purpose, an infinite periodic lattice of elements is introduced, coinciding in some region of space with the one under study. Modeling of a single element of such a lattice with imposed periodic boundary conditions allows us to construct dispersion characteristics and determine the effective sound velocity in this region. However, the effective impedance of the medium remains unknown, which, as a result, does not exclude the need for FEM-type methods. It should also be noted that the realization of the obtained parameters is difficult due to their high anisotropy [14, 15].

Another solution is related to the application of scattering theory methods to metamaterials [16-22]. Even before the active discussion of metamaterials in the literature, these approaches were actively used to analyze wave propagation in regular and randomly inhomogeneous media [23, 24]. In this approach, each point of the medium is considered as a scatterer that emits secondary waves in response to the field of the original incident wave and the fields scattered by other elements. Mathematically, the result of the calculation turns out to be equivalent to solving the original acoustic equations. In the case when the metamaterial is represented as separate elements placed in the background medium, each of them can be considered as a single scatterer characterized by scattering coefficients [25, 26]. Since the number of such scatterers is much smaller than the number of elements when discretizing the medium, the method turns out to be computationally efficient.

In the present work, we propose to use this approach and perform calculations in two steps, separately considering the processes of multiple field scattering inside a single element of a metamaterial and inside a lattice of discrete scatterers. Depending on the order of steps, this makes it possible either to calculate the field inside a metamaterial with a given structure of its elements or to solve the inverse problem of finding such a structure.

DISCRETE EQUATION OF LIPPMANN-SCHWINGER TYPE

Let the metamaterial be a system of a finite number of discrete elements of small size placed in a liquid homogeneous background medium. The acoustic field with time dependence $\sim \exp(-i\omega t)$ is created by primary sources located in the region X outside the metamaterial. For each q -element, a circle (in the two-dimensional case, space dimension $d = 2$) or a ball (in the three-dimensional case, $d = 3$) of Γ_q minimum possible radius centered at a point $\vec{r}_q$ is constructed containing it. We consider a sufficiently wide class of metamaterials for which these circles (or balls) have no common points. There are no requirements that the elements of the metamaterial be arranged periodically or be identical. Further reasoning is carried out only for the two-dimensional case; similar results are obtained for the three-dimensional case.

We consider an arbitrary fixed element of the metamaterial with index q . The region of space $\overline{\Gamma} \equiv \left(R^d \setminus X\right) \setminus \bigcup_q \Gamma_q$ is occupied by a homogeneous background medium with wave number k_0 and contains no sources. The acoustic pressure field $p(\vec{r})$ in this region can be represented as

$$p(\vec{r}) = p_0(\vec{r}) + \sum_{t \neq q} p_{\text{sc}}^{(t)}(\vec{r}) + p_{\text{sc}}^{(q)}(\vec{r})\,;\ \vec{r} \in \overline{\Gamma}\,. \tag{1}$$

Here $p_0(\vec{r})$ is the field of primary sources; $\sum_{t \neq q} p_{\text{sc}}^{(t)}(\vec{r})$ is the sum of the fields scattered on all elements of the metamaterial except q -th, and $p_{\text{sc}}^{(q)}(\vec{r})$ is the field scattered on q -th element of the metamaterial. The value $p_{\text{inc}}^{(q)}(\vec{r}) \equiv p_0(\vec{r}) + \sum_{t \neq q} p_{\text{sc}}^{(t)}(\vec{r})$ represents the field incident on q -th element of the metamaterial. Taking into account that its sources are located outside Γ_q , and the sources $p_{\text{sc}}^{(q)}(\vec{r})$ - inside Γ_q , introducing a cylindrical coordinate system with the origin at the point $\vec{r}_q$ and some fixed direction of the polar axis, we can write the expansions of these fields into series on cylindrical functions:

$$\begin{aligned} p_{\text{inc}}^{(q)}(\vec{r}) &= -\frac{i}{4} \sum_{n=-\infty}^{\infty} \exp\left[in\theta_q(\vec{r})\right] \cdot A_n^{(q)} \cdot J_n\left(k_0\left|\vec{r} - \vec{r}_q\right|\right); \\ p_{\text{sc}}^{(q)}(\vec{r}) &= -\frac{1}{16} \sum_{n=-\infty}^{\infty} \exp\left[in\theta_q(\vec{r})\right] \cdot B_n^{(q)} \cdot H_n^{(1)}\left(k_0\left|\vec{r} - \vec{r}_q\right|\right). \end{aligned} \tag{2}$$

Here n is the multipole order; $\theta_q(\vec{r})$ is the angle that the vector $\vec{r} - \vec{r}_q$ makes with the polar axis of the coordinate system; $J_n(\cdot)$ and $H_n^{(1)}(\cdot)$ are the Bessel and Hankel functions of the first kind n -of order, respectively. The coefficients $A_n^{(q)}$ and $B_n^{(q)}$ form vectors $\vec{A}^{(q)}$ and $\vec{B}^{(q)}$, which are related to each other in a linear way. To describe this relationship, the matrix $\hat{T}^{(q)}$ [25 -29] is introduced:

$$\vec{B}^{(q)} = 4i \cdot \hat{T}^{(q)} \cdot \vec{A}^{(q)}. \tag{3}$$

It fully characterizes the properties of the metamaterial element as an acoustic field scatterer. Calculation of the matrix $\hat{T}^{(q)}$ may present a significant difficulty, since, strictly speaking, it (as well as vectors $\vec{A}^{(q)}$ and $\vec{B}^{(q)}$) contains an infinite number of elements. However, for small wave size scatterers, in most cases it is sufficient to consider only the monopole and dipole order of scattering [25]. If, in addition, the scattered field does not change when the metamaterial element is rotated, the matrix $\hat{T}^{(q)}$ takes a diagonal form:

$$\hat{T}^{(q)} = -\frac{i}{4}\mathrm{diag}\left(\beta_1^{(q)} \quad \beta_0^{(q)} \quad \beta_1^{(q)}\right), \tag{4}$$

where $\beta_0^{(q)}$ and $\beta_1^{(q)}$ are the monopole and dipole scattering coefficients, respectively. Taking into account the identity $J_{-n}(\cdot) = (-1)^{-n} J_n(\cdot)$ and the fact that Γ_q and Γ_t have no common points at $q \neq t$, using the graph addition theorem for an arbitrary cylindrical function $\vartheta_n(\cdot)$ we can obtain the expression

$$\vartheta_n\left(k_0\left|\vec{r} - \vec{r}_q\right|\right) \exp\left[in\theta_q(\vec{r})\right] =$$
$$= \sum_{m=-\infty}^{\infty} J_m\left(k_0\left|\vec{r} - \vec{r}_t\right|\right) \exp\left[im\theta_t(\vec{r})\right](-1)^{n-m}\vartheta_{n-m}\left(k_0\left|\vec{r}_q - \vec{r}_t\right|\right) \exp\left[i(n-m)\theta_t(\vec{r})\right]$$

$$, \vec{r} \in \overline{\Gamma}, \tag{5}$$

For the field $p_0(\vec{r})$ then a decomposition similar to (2) is valid:

$$p_0^{(q)}(\vec{r}) = -\frac{i}{4}\sum_{n=-\infty}^{\infty} \exp\left[in\,\theta_q(\vec{r})\right]\cdot A_{n,0}^{(q)} \cdot J_n\left(k_0\left|\vec{r}-\vec{r}_q\right|\right). \qquad (6)$$

Then, taking into account (5), expressions (1) - (3) and (6) are reduced to the equation

$$\vec{A}^{(q)} = \vec{A}_0^{(q)} + 4i\sum_t \hat{G}(\vec{r}_q,\vec{r}_t)\hat{T}^{(t)}\vec{A}^{(t)}. \qquad (7)$$

Here we introduce the matrix $\hat{G}(\vec{r}_q,\vec{r}_t)$, whose elements are equal to 0 at $q=t$ and equal to

$$G_{mn}(\vec{r}_q,\vec{r}_t) = -\frac{i}{4}(-1)^{n-m} H_{n-m}^{(1)}\left(k_0\left|\vec{r}_q-\vec{r}_t\right|\right)\exp\left[i(n-m)\theta_t(\vec{r}_q)\right] \text{ at } .q \neq t$$

The equation (7) is a discrete Lippmann-Schwinger type equation describing the processes of multipole field scattering by metamaterial elements. The vector $\vec{A}^{(q)}$ contains the set of coefficients of the multipole expansion of the field in the coordinate system with origin at the point $\vec{r}_q$. Its multiplication by the matrix $\hat{T}^{(q)}$ gives the vector $\vec{B}^{(q)}$ of the scattered field expansion coefficients. The matrix $\hat{G}(\vec{r}_q,\vec{r}_t)$ plays the role of Green's function. It is zero if its arguments coincide. This illustrates the fact that the field scattered on a metamaterial element does not directly affect it; the totality of multiple scattering processes within the element is accounted for by the matrix .$\hat{T}^{(q)}$

If the matrices $\hat{T}^{(q)}$ are known for each element of the metamaterial, equation (7) is solved with respect to the vector $\vec{A}^{(q)}$. Then the field $p(\vec{r})$ is defined using expressions (2) and (3). Conversely, equation (7) can be considered with respect to the unknown matrices $\hat{T}^{(q)}$, assuming the field $p(\vec{r})$ is known. In each case, the problem is divided into two steps: consideration of acoustic field scattering by a single element of the metamaterial and consideration of multiple scattering between individual elements. In the second step, it is sufficient to specify only a few scattering coefficients and scatterer coordinates. The number of variables in this case can be much smaller than when solving the problem using the FEM.

The equation (7) remains valid regardless of the particular arrangement of the metamaterial elements, since only the fields in the region $\overline{\Gamma}$ are considered. The processes inside Γ_q may not be described by the Helmholtz equation, or their nature may be non-acoustic at all. It should be noted once again that the possibility of covering the metamaterial elements with a set of circles (or spheres) Γ_q is a significant limitation of the proposed method.

DESIGN OF INSULATING NON-REFLECTIVE SHELL

In the simplest case, the metamaterial element is described by a single monopole scattering coefficient $\beta_0 \equiv |\beta_0| \exp(i\psi_0)$. For a passive medium without absorption, its absolute value $|\beta_0|$ and phase ψ_0 are not arbitrary but are related by the relation $|\beta_0| = -4\sin\psi_0$ [, ,252630 -32]; for active metamaterials there are no such restrictions. In the case when all other elements of the matrices $\hat{T}^{(q)}$ are zero, the equation (7) is scalar:

$$A_n^{(q)} = A_{n,0}^{(q)} + \sum_{t \neq q} G_{00}(\vec{r}_q, \vec{r}_t) \beta_0^{(q)} A_n^{(q)} \text{ при } n = 0; \; A_n^{(q)} = A_{n,0}^{(q)} \text{ при } n \neq 0. \quad (8)$$

The matrix element $G_{00}(\vec{r}_q,\vec{r}_t) = -\frac{i}{4} H_0^{(1)}\left(k_0\left|\vec{r}_q - \vec{r}_t\right|\right)$ here represents the delayed Green's function of the Helmholtz equation.

As an illustration, we consider the problem of designing an annular shell made of a metamaterial (Fig. *1a*). Its properties are such that, if the source is inside the shell, its field inside the shell is not distorted by it and does not pass outside. The field of the source located outside does not pass inside and is minimally scattered by the shell. The last circumstance relates the set problem to the problem of concealment [4 -7]. The described requirements can be written using the Green's function $G(\vec{r}_1,\vec{r}_2)$ of an inhomogeneous medium:

$$\begin{array}{ll} G(\vec{r}_1,\vec{r}_2) = G_{00}(\vec{r}_1,\vec{r}_2) & \text{при } |\vec{r}_1| < R_1, |\vec{r}_2| < R_1 \text{ или } |\vec{r}_1| > R_2, |\vec{r}_2| > R_2; \\ G(\vec{r}_1,\vec{r}_2) = 0 & \text{при } |\vec{r}_1| < R_1, |\vec{r}_2| > R_1 \text{ или } |\vec{r}_1| > R_2, |\vec{r}_2| < R_2. \end{array} \quad (9)$$

The equation (8) under the conditions (9) can be solved in different ways [,3334]. In this case, a method similar to the one described in [35] was used. It consists in using the Born approximation and iterative refinement of the parameters of the medium inside the shell. It was assumed that it consists of 30 annular layers with the same elements in each layer.

After the scattering coefficients of the shell elements were determined, the shell was tested. In Fig. *1b*, the solid lines represent the acoustic pressure from the source located at S_1 (inside the shell) or from the source at S_2 (outside). For comparison, the dashed lines show similar dependencies in the absence of the shell. It can be seen that the source field S_1 is not distorted by the shell and is equal to zero outside it, i.e. the first requirement is fulfilled with great accuracy. The field of the source S_2 does not penetrate inside the shell, but is scattered by it. It is especially noticeable from the side opposite to the source: here a "shadow" zone is formed. Thus, the condition of concealment is not completely fulfilled. This is due to the fact that, in contrast to the works [,45], the elements of the calculated structure are isotropic, and in contrast to the works [,67], the external field does not penetrate inside the shell.

Fig. *1c* on the complex plane shows the values of the found coefficients β_0 for different elements of the shell. The circle Ω with the center in the point $-2i$ and radius 2 denotes the set of scatterers for which the law of conservation of energy is satisfied [,3132]. The points lying inside Ω , correspond to scatterers with absorption, and outside to those containing an additional energy source. Although it is in principle possible to limit the iterative solution of the equation (8) to the case of energy conservation in scattering, this leads to a significant deterioration of the obtained solution. Thus, it is reasonable to realize the calculated design exactly in the class of active metamaterials.

TUNABLE LENS DESIGN

The equations (7), (8) can be used to calculate metamaterials that change their wave properties under external influence. As an illustration, we consider the problem of designing a planar lens with a tunable focus. Such a lens (Fig. *2a*) is a lattice of $N \cdot (2M + 1)$ elements of a metamaterial whose centers are located at the points with coordinates ; ; $\vec{r}_{nm} \equiv \{x_n; y_m\}$ $0 \le n < N - M \le m \le M$. The tuning is realized by a slight compression or stretching of the lattice along one of the coordinate axes.

To simplify the practical realization, all elements are chosen identical. Their monopole and dipole scattering coefficients are β_0 and β_1, respectively. Along the axis OX, they are spaced uniformly with a step $s_x a$, i.e., $x_n = s_x na$. The step along the axis OY, equal to $s_y b_m = y_{m+1} - y_m$, is non-uniform. Here, the values a and b_m represent the base distances between the centers of adjacent elements, and the coefficients s_x and s_y, which are close to unity, determine the stretching or compression of the lens along the corresponding axis.

Thus, when N and M are fixed, the design is fully described by the parameters , ,β_0 β_1 a and b_m. In determining their values, the following considerations are taken into account. First, the focal length of the designed lens should change significantly when changing coefficients s_x or s_y within a few percent, since such a small change can be relatively easy to realize in practice. Second, the acoustic field reflected from the lens should be minimized as much as possible. Third, the properties of the lens should remain stable over some frequency band. To take into account the last circumstance, it is necessary to specify their frequency derivatives along with the scattering coefficients, which complicates the analysis. On the other hand, for general considerations, scattering coefficients with values close to $-4i$, since this point corresponds to resonance [, ,253132], can be excluded from consideration. Ultimately, it is advisable to perform such a check after the internal structure of the metamaterial element has been determined based on the values of the scattering coefficients.

As a result of the solution of the described problem at $M = N = 10$, the values of scattering coefficients $\beta_0 = -0.587 + 0.088i$ and $\beta_1 = 0.497 + 0.063i$ were obtained; the step a is equal to $0.257\lambda_0$, where $\lambda_0 \equiv 2\pi/k_0$ is the wavelength in the homogeneous background medium surrounding the lens. The dependence of the steps b_m on the index m is close to quadratic: $b_m = 0.192\lambda_0 + 0.00019\lambda_0(m-1)^2$. Such a lens has thickness $2.57\lambda_0$ and transverse dimension $4\lambda_0$. In the simulation, the incident acoustic field was radiated by a plane source of width $8\lambda_0$, located at a distance $2\lambda_0$ from the lens. This source creates a wave beam in a homogeneous background medium, the amplitude of which at the origin is equal to . p_0

In Fig. *2b*, the black lines show the results of calculating the amplitude of the acoustic pressure field $p(x)$ on the OX axis to the right of the lens, normalized to p_0 . It can be seen that when the lens is transversely deformed, when the value of s_y varies within $\pm 5\%$, the area of focus moves significantly and, thus, the described configuration does solve the problem. At the same time, the field amplitude in the focal drag region changes within 10%, i.e., relatively weakly. The modeling also showed that changing the value of s_x has little effect on the amplitude and position of the focus.

For practical realization of a lens with the described parameters it is necessary to propose a design of a metamaterial element possessing the found scattering coefficients. In the general case, this represents a separate difficult problem. One of the possible methods of its solution is to consider a multilayer elastic cylinder as such an element. The thicknesses and materials of its layers are selected so as to minimize the difference of the scattering coefficients from the required ones. For the obtained values of the scattering coefficients, if water is used as a background medium, a good match can be obtained using homogeneous steel cylinders with radius $0.077\lambda_0$. This value is less than half of the minimum distance between the centers of the metamaterial elements, which makes it possible to implement it in practice.

To check the performance of the lens in the broadband mode, the frequency of the used radiation was reduced by 10% at fixed geometrical dimensions of all elements. The calculated dependence of the pressure $p(x)$ on the axis in this case at $s_y = 1$ is represented in Fig. *2b* by a gray line. It can be seen that its difference from the black line drawn at the initial radiation frequency is small. Consequently, the calculated lens can be used also when working with nonmonochromatic signals.

CONCLUSION

The methods of scattering theory can be successfully used to consider metamaterials consisting of individual elements placed in a background medium. They allow, to calculate the acoustic field inside a metamaterial with a given structure or, conversely, to determine this structure if it is known how the field is to be transformed. The proposed two-step method allows us to break each of these problems into two parts, which can significantly speed up the solution. Such a breakdown is possible because the metamaterial elements have a small wave size, and it is sufficient to specify only a few scattering coefficients to describe the scattering on them.

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FIGURE CAPTIONS

Figure 1. Schematic of a cylindrical shell made of metamaterial. At the points S_1 and S_2 the sources are located (a). View of the acoustic pressure profile along the line $y = x$, when the cylindrical shell is present (solid lines) or absent (dashed lines). Black lines correspond to the location of the source at S_1; gray lines correspond to the location at S_2. The gray bars indicate points inside the envelope (b). Calculated values of the scattering coefficient β_0 on the complex plane for different elements of the metamaterial inside the shell. The black line represents the circle Ω (c).

Figure 2. Schematic representation of a lens made of a metamaterial. The dots represent the centers of its elements. The acoustic wave is generated by a plane source located to the left of the lens (a). Spatial distribution of the acoustic pressure amplitude on the lens axis to the right of the lens at different values of the stretching coefficient s_y. Black lines correspond to the initial frequency of radiation; gray line - to the frequency of radiation reduced by 10% (b).

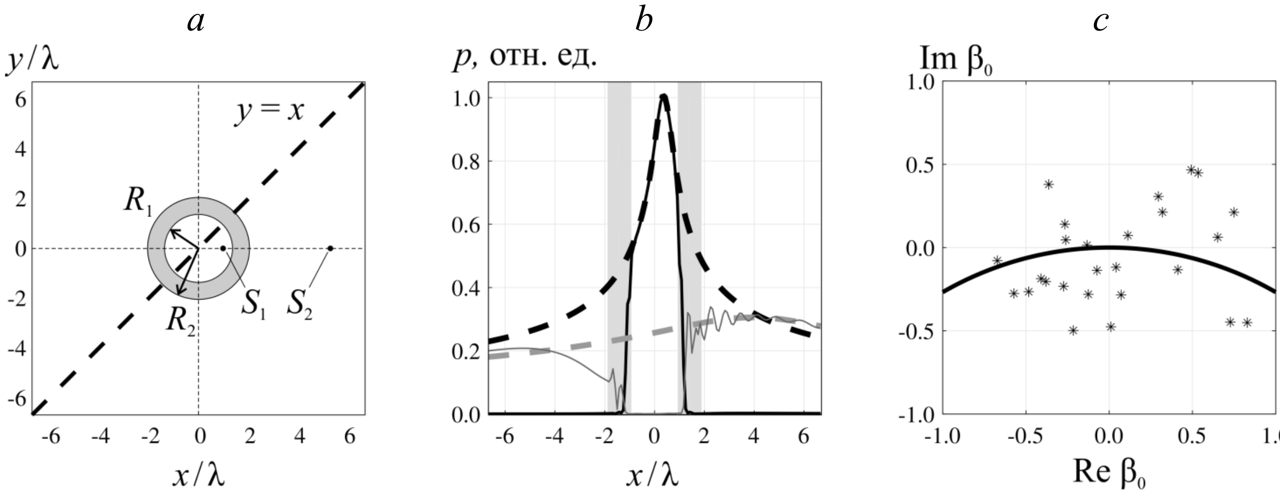

Fig. 1.

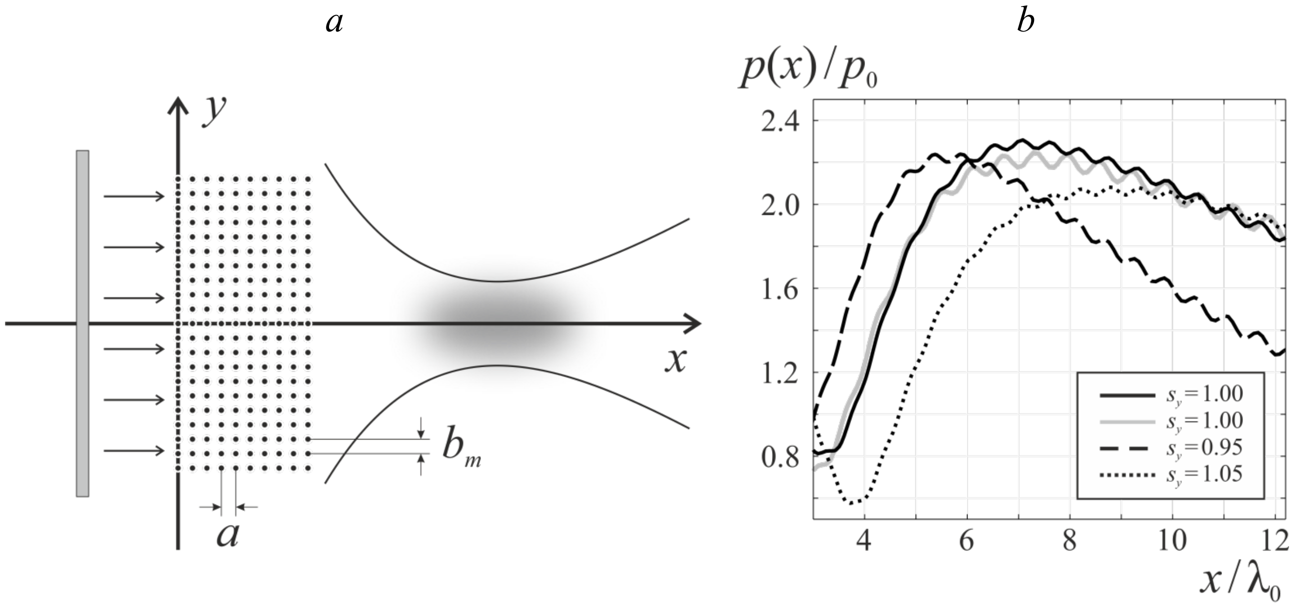
a
b
y
x
b_m
a
$p(x)/p_0$
2.4
2.0
1.6
1.2
0.8
4
6
8
10
12
x/λ_0
$s_y = 1.00$
$s_y = 1.00$
$s_y = 0.95$
$s_y = 1.05$

Fig. 2.