(2+1)D SOLITON PAIRS IN A PLANAR QUADRATIC NONLINEAR CRYSTAL WITH INHOMOGENEITY

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Abstract. The process of propagation and formation of pulse pairs in a quadratically nonlinear crystal with two waveguides is investigated when parameters related to the position of the waveguides relative to each other, delay and phase ratio between pulses change. A change in the pulse propagation mode during the approach of waveguides and the dependence of the nature of the interaction between the pulses on the initial phase ratio were found.

Keywords: *optical solitons, quadratic nonlinearity, two-dimensional pulses, inhomogeneity, nonlinear optics*

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INTRODUCTION

One of the most intriguing areas of optical research is nonlinear optics, where solitons occupy a special place. An optical soliton is a solitary laser pulse of a certain duration (from nano- to femtoseconds) possessing a carrier frequency of the visible range and capable of propagating in a nonlinear dispersing medium without changing its shape over long distances. Of particular interest are optical solitons in the Kerr medium, which is described by the nonlinear Schrödinger equation (NSE) possessing a soliton solution [1]. Light bullets or multidimensional solitons in a homogeneous Kerr medium experience self-focusing collapse when a certain threshold related to the pulse amplitude is exceeded. In particular, two-dimensional NUS leads to the so-called Townes soliton [2], which is degenerate in free space in the sense that it occurs at only one value of energy. From a physical point of view, the Townes soliton is an unstable state that shares two modes of light propagation: the blurring of the pulse-beam caused by diffraction and its unbounded self-focusing due to nonlinearity [3,4]. This shows that nonlinearity is important for the formation of solitons but does not guarantee their stability.

Many configurations of optical media in which solitons are stable have been found. These configurations include materials accounting for higher orders of nonlinearity and dispersion [5], media with a combined type of nonlinearity [6], and media with inhomogeneities that can compensate for diffraction blurring [7]. Solitons have also been obtained in an artificial optical medium described by the fractional Schrödinger equation [8]. Some results related to the consideration of fractional media are the subject of a mini-review [9]. Since unbounded self-focusing is the main obstacle to the formation of solitons in the Kerr medium, quadratic nonlinear media [10], in which collapse is absent, have also been considered. Another interesting approach to soliton generation is to take into account the coupling dispersion between planar waveguides with Kerr nonlinearity as an analog of spin-orbit coupling in a Bose-Einstein condensate [11].

In our work, we considered a quadratically nonlinear medium with two planar waveguides. By varying the parameters related to the position and size of the waveguides, as well as by varying the parameters of the trial solution, we wanted to achieve stable propagation of the pulse pair over long distances or, in other words, to obtain a soliton solution. As mentioned above, we consider the propagation of a pair of pulse beams whose initial parameters can be controlled. The consideration is carried out in the second harmonic generation (SHG) regime under normal dispersion, where each pulse from the trial solution is assigned its own waveguide with specially defined characteristics. The coupling between them is accomplished by a fraction of the pulse-beam energy that can penetrate the region between the waveguides. This work is based on the results of earlier studies [12,13].

BASIC EQUATIONS

The description of the process of generation of the second optical harmonic is carried out in the quasi-optical approximation with the dependence of the linear susceptibility of the medium on the coordinate $\chi_\omega(r_\perp)$ in the form: $\chi_\omega(r_\perp) = \chi_\omega^{(0)}[1 + f_\omega(r_\perp)]$, where $r_\perp$ is the radius-vector perpendicular to the central axis of the waveguide, $\chi_\omega^{(0)}$ is the linear susceptibility of the medium at the center of the cross-section of one of the waveguides, and $f_\omega(r_\perp)$ is a dimensionless function that describes the profile of the waveguide. In this case, the system of equations to describe the GWG process in a system of two planar waveguides takes the form:

$$i\frac{\partial A_1}{\partial z} + \frac{\beta_\omega}{2}\frac{\partial^2 A_1}{\partial \tau^2} - \alpha_\omega A_1^* A_2 e^{i(2k_1-k_2)z} = \omega g_1(x)A_1 + \frac{c}{2n_\omega^{(0)}\omega}\frac{\partial^2 A_1}{\partial x^2}, \qquad (1)$$

$$i\frac{\partial A_2}{\partial z}+\frac{\beta_{2\omega}}{2}\frac{\partial^2 A_2}{\partial \tau^2}-\alpha_{2\omega}A_1^2 e^{-i(2k_1-k_2)z}=2\omega g_2(x)A_2+\frac{c}{4n_{2\omega}^{(0)}\omega}\frac{\partial^2 A_2}{\partial x^2}.$$

Here $A_{1,2}$ is the envelope amplitude of the first and second harmonic, respectively, τ is the local time, which is $\tau=t-\frac{z}{2}\left(\frac{1}{v_g^{(\omega)}}+\frac{1}{v_g^{(2\omega)}}\right)$, where t is time, z is the propagation direction, x is the transverse coordinate, $v_g^{(\omega,2\omega)}$ is the group velocities for the corresponding harmonics at the center of the waveguide, where $\left|v_g^{(2\omega)}-v_g^{(\omega)}\right|<<v_g^{(2\omega)},v_g^{(\omega)}$. The coefficient $\beta_{\omega,2\omega}$ is responsible for the dispersion of the group velocity at the center of the waveguide, the nonlinearity effects are responsible for $\alpha_\omega=\frac{2\pi\omega}{cn_\omega^{(0)}}\chi^{(2)}(2\omega,-\omega)$ and $\alpha_{2\omega}=\frac{4\pi\omega}{cn_{2\omega}^{(0)}}\chi^{(2)}(\omega,\omega)$, where , $\chi^{(2)}(2\omega,-\omega)\chi^{(2)}(\omega,\omega)$ are the coefficients of the second-order nonlinear susceptibility at the center of the waveguide. The wave numbers for the first second harmonic are given by $k_1=k(\omega)$ and $k_2=k(2\omega)$. Diffraction is described by the second summand in the right-hand side of each equation, where $n_{\omega,2\omega}^{(0)2}(x)=1+\left(n_{\omega,2\omega}^{(0)2}-1\right)\left(1+f_{\omega,2\omega}(x)\right)$. - are the refractive indices of the harmonics, c is the speed of light in vacuum. The first summand in the right part of both equations is responsible for the influence of inhomogeneity, where : $g_{1,2}(x)$

$$g_1(x)=\frac{n_\omega^{(0)2}-1}{2cn_\omega^{(0)}}f_\omega(x), \qquad g_2(x)=\frac{n_{2\omega}^{(0)2}-1}{2cn_{2\omega}^{(0)}}f_{2\omega}(x),$$

where $f_{\omega,2\omega}(x)$ is a one-dimensional dimensionless function defining the waveguide profile. In the following we will consider the case of group and phase synchronism, so $v_g=v_g^{(2\omega)}=v_g^{(\omega)}$ and . $2k_1=k_2$

To perform numerical simulations, the system needed to be depersonalized, so the following dimensionless parameters were introduced: $A_{1,2} = \bar{A}_{1,2}A_{in}$, , , , ,$z = \overline{z}l_{nl}x = \overline{x}R_{in}\tau = \overline{\tau}\tau_{in}\Delta\bar{k} = \Delta k l_{nl} l_{nl} = (\alpha_{\omega}A_{in})^{-1}a_{\omega,2\omega} = R_{in}\bar{a}_{\omega,2\omega}A_{in}$. ,is the peak amplitude at the center of one of the waveguides,L_{nl} is the nonlinear length equal to the distance at which full energy transfer between harmonics occurs,R_{in}, τ_{in} is the initial radius and duration of the beam pulse. Dimensionless expressions for the coefficients from (1) responsible for diffraction, dispersion, inhomogeneity, and nonlinearity were also introduced: $D_{x1} = \frac{cl_{nl}}{2\omega n_{\omega}^{(0)}R_{in}^2}$, , , , ,$D_{x2} = \frac{cl_{nl}}{4\omega n_{2\omega}^{(0)}R_{in}^2}D_{\tau1} = \frac{\beta_{\omega}l_{nl}}{2\tau_{in}^2}D_{\tau2} = \frac{\beta_{2\omega}l_{nl}}{2\tau_{in}^2}D_{q1} = \frac{2\pi\omega l_{nl}}{cn_{\omega}^{(0)}\bar{a}_{\omega}^2}\chi_{\omega}^{(0)}D_{q2} = \frac{4\pi\omega l_{nl}}{cn_{2\omega}^{(0)}\bar{a}_{2\omega}^2}\chi_{2\omega}^{(0)}\gamma = \frac{\alpha_{2\omega}}{\alpha_{\omega}}$. ,Introducing, the above dimensionless parameters, we obtain:

$$i\frac{\partial\bar{A}_1}{\partial\bar{z}} = D_{q1}p_1(\bar{x})\bar{A}_1 - D_{\tau1}\frac{\partial^2\bar{A}_1}{\partial\bar{\tau}^2} + \bar{A}_1^*\bar{A}_2 + D_{x1}\frac{\partial^2\bar{A}_1}{\partial\bar{x}^2},$$
$$i\frac{\partial\bar{A}_2}{\partial\bar{z}} = D_{q2}p_2(\bar{x})\bar{A}_2 - D_{\tau2}\frac{\partial^2\bar{A}_2}{\partial\bar{\tau}^2} + \gamma\bar{A}_1^2 + D_{x2}\frac{\partial^2\bar{A}_2}{\partial\bar{x}^2}. \tag{2}$$

$$p_{1,2} = a_{\omega,2\omega}^{-2}\left[1 - exp\left(-\frac{(x-x_w)^2}{a_{\omega,2\omega}^2}\right) - exp\left(-\frac{(x+x_w)^2}{a_{\omega,2\omega}^2}\right)\right]. \tag{3}$$

The trial solution that was fed to the input of the medium is as follows:

$$A_1(z=0) = E_{11}\,exp(-(x-x_w)^2 - (\tau-\tau_{10})^2 + i\varphi_{11})$$
$$+ E_{12}\,exp(-(x+x_w)^2 - (\tau-\tau_{20})^2 + i\varphi_{12})$$
$$A_2(z=0) = E_{21}\,exp(-(x-x_w)^2 - (\tau-\tau_{10})^2 + i\varphi_{21})$$
$$+ E_{22}\,exp(-(x+x_w)^2 - (\tau-\tau_{20})^2 + i\varphi_{22}), \tag{4}$$

Here$E_{11,12}$ and$E_{21,22}$ define the initial amplitude values for the beams at the fundamental and doubled frequencies, respectively. Parameters$\varphi_{11,12}$ and$\varphi_{21,22}$ - define the initial phase ratio between the pulses, and$\tau_{10,20}$ for the time delay between them. The parameterx_w , which is also included in the expression for the waveguide profile function (3), is responsible for the position of the waveguide centers.

The waveguide function view (3) provides the waveguide refractive index minima near$x = \pm x_w$. The optical beam is trapped near the center of the waveguide, but its tails penetrate the zone between them, providing coupling of pulse-beam pairs. The characteristic width of the waveguides is $a_{\omega,2\omega}$.

Numerical calculations are based on the method developed in [12], which ensures that the integrals of motion inherent in the system of equations (2) are preserved. Checking the conservation of integrals during the calculation guarantees the accuracy of the results.

NUMERICAL SIMULATION RESULTS

To investigate the process of pulse-beam formation and propagation in a quadratically nonlinear medium with two planar waveguides, we numerically simulated system (2) with boundary condition (4). The propagation of pulses occurred in the GWG regime, when $E_{21,22} = 0$, i.e., only beams of fundamental frequency were fed to the input. In order to follow the effect of the initial phase ratio on the propagation of the pulse-beam pair, we varied $\varphi_{11,12}$, and the signals could have both the same phase($\varphi_{11} = \varphi_{12} = 0$) , and different phases whenφ_{12} was smoothly varied between 0 and π. In addition, the signals could have a time delay between each other when $\tau_{10} \neq \tau_{20}$.

The diffraction coefficients were taken as ,$D_{x1} = 0.1 D_{x2} = 0.05$. Dispersion coefficients were equal to ,$D_{\tau1} = 0.05 D_{\tau2} = 0.1$, which corresponds to normal dispersion ($D_{\tau 1,2} > 0$). The nonlinearity coefficient was assumed to be$\gamma = 0.5$. The parameters responsible for characteristics of the waveguides were assumed to be equal to , ,$a_\omega = a_{2\omega} = 2D_{q1} = 10D_{q2} = 10$. The parameter responsible for the position of the waveguide centersx_w , could be changed during the calculation. The values of dimensionless parameters that set the characteristics of the modeled medium remained constant for all calculations.

Based on the results of [13], a soliton-like solution for a pair of pulse-beams was obtained. This solution is not exactly soliton-like, since it does not have constant characteristics, but changes its spatial and temporal dimensions and intensity periodically, which is sometimes called "breathing". However, it is localized, since most of the intensity is confined in a small region of space-time.

The soliton pair is formed not immediately, but only after the completion of the process of energy transfer between the main and second harmonics in each waveguide, which approximately corresponds to the passage of a distance equal to 20 nonlinear lengths by the pair of signals. The resulting "breathing" pulse pair propagated over a distance of 600 nonlinear lengths with preservation of the spatiotemporal shape, which can be observed in Fig. *1a*. The signal profile at the left boundary of the modeled crystal retains its shape at the end of the propagation distance, but loses in intensity, which can be seen in Fig. *1g-1e*. However, the main loss of intensity occurs at the initial stage, when the energy is pumped into the second harmonic, and in addition, a part of the energy not yet captured in the soliton moves away from the main beam. A comparison of the profiles at the distances $z = 100$ and $z = 200$ (Fig. 1 e,f) shows that at this stage the peak intensity not only did not drop, but also slightly increased due to focusing.

Analysis of the peak intensity in Fig. *1a* shows that this soliton-like solution generally persists up to $z = 600$.

It is also worth noting that in the region between the waveguides a non-zero intensity level is observed over the entire propagation distance. As mentioned earlier, part of the energy seeps through the waveguide walls, which indicates that the pulses in the individual waveguides interact with each other. Figs. *1b* and *1c* show the dependence of the position of the spatial and temporal centers of the pulse-beam pair on the propagation distance. It can be seen that in Fig. *1c* the position of the pulse maxima along the axis up to$x$ oscillates with a small amplitude near the center of the waveguide. The temporal maxima along with propagation move away from the given initial position, which means that the pulses repel each other.

Fig. 2 shows the calculation data for a pair of pulses launched with a delay $\tau_{10} = 0.5, \tau_{20} = -0.5$, where the boundary condition gradually changes the phase of the pulse in the second waveguideφ_{12} in the range from 0 to π in steps of 0.1π. The phase change does not affect the propagation distance, the pulse pair propagates stably for 600 nonlinear lengths, as in the first calculation for signals with the same phase. The effect of the initial phase ratio is to change the nature of the motion of the time centers of the pulses as well as the nature of their interaction, which can be seen in Figs. *2a* and *2b*. When gradually increases the phase φ_{12} from 0 to π, the pulse pair gradually moves from mutual repulsion at $\varphi_{11} = \varphi_{12} = 0$ to mutual attraction at $\varphi_{11} = 0, \varphi_{12} = \pi$. In the intermediate regime at $\varphi_{12} \cong 0.6\pi$, the time center oscillates around the equilibrium position until$z = 300$. After that, depending on the initial phase ratio and the phase value of each pulse, the time centers of the pulses at the fundamental frequency begin to repel, as in the case with the same phase, but in this case the movement of the centers occurs at different speeds (Fig. *2c*). In the signal with a smaller initial phase, the center moves faster, while the center of the second pulse moves slower. The moment of the beginning of the movement of the pulse centers along the time axis coincides with the end of the half-period of the energy transfer process between the pulses, which became especially pronounced at the selected phase ratio ($\varphi_{11} = 0, \varphi_{12} = 0.6\pi$) , which can be seen in Fig. *2c* and *2d*. It is worth noting that the time delay was introduced so that the effect of attraction and repulsion of the time centers of the pulses could be clearly observed. When the pulses are launched synchronously, the effect is preserved, but for the distances considered, the deviation of the pulse centers is very small and will become noticeable only at a sufficiently large propagation distance.

The results obtained when investigating the influence of the waveguide position on the propagation mode of the soliton-like solution are shown in Fig. 3. By changing the parameter x_w , which is included both in the initial solution (4) and in the dimensionless function defining the waveguide profile (3), we changed the position of the centers of the gradient waveguides. In Fig. 3 we can see that at parameter $x_w = 1.9$ ($a_{\omega,2\omega} = 2$) the waveguides start to partially overlap, which increases the influence of pulses in different waveguides on each other. This is manifested in the increased amplitude of the oscillation of the spatial centers of the pulses compared to the results for non-overlapping waveguides at $x_w = 2.0$. The energy of the pulse "tails" that previously leaked into the region between the waveguides begins to accumulate in the overlapping region, which can be seen in Fig. *3c* in the plots for signal profiles at different values of propagation distance z. With further convergence of the waveguides, the amplitude of oscillations of the spatial centers continues to increase, and when reaching $x_w = 1.7$, the soliton-like mode is broken: the pulse pair decays at a distance of 40 nonlinear lengths.

CONCLUSION

Thus, the process of formation and propagation of a soliton-like solution in a pair of coupled optical waveguides in a planar quadratic-nonlinear crystal under changing parameters related to the position of the waveguide centers and the phase ratio between pulses is considered. It was found that the character of pulse interaction depends on the initial phase ratio. At a certain initial phase difference ($\delta\varphi = 0.6\pi$), energy pumping between the waveguides is observed, which takes about 600 nonlinear lengths. Overlapping of the waveguides with each other ($x_w = 1.9$) enhances the coupling between the waveguides, which also affects the spatial position of the signals during propagation, adding a noticeable oscillation of the pulse centers along the spatial axis. The peak intensity decreases as some of the pulse energy begins to leak into the region of overlap between the waveguides, lingering there.

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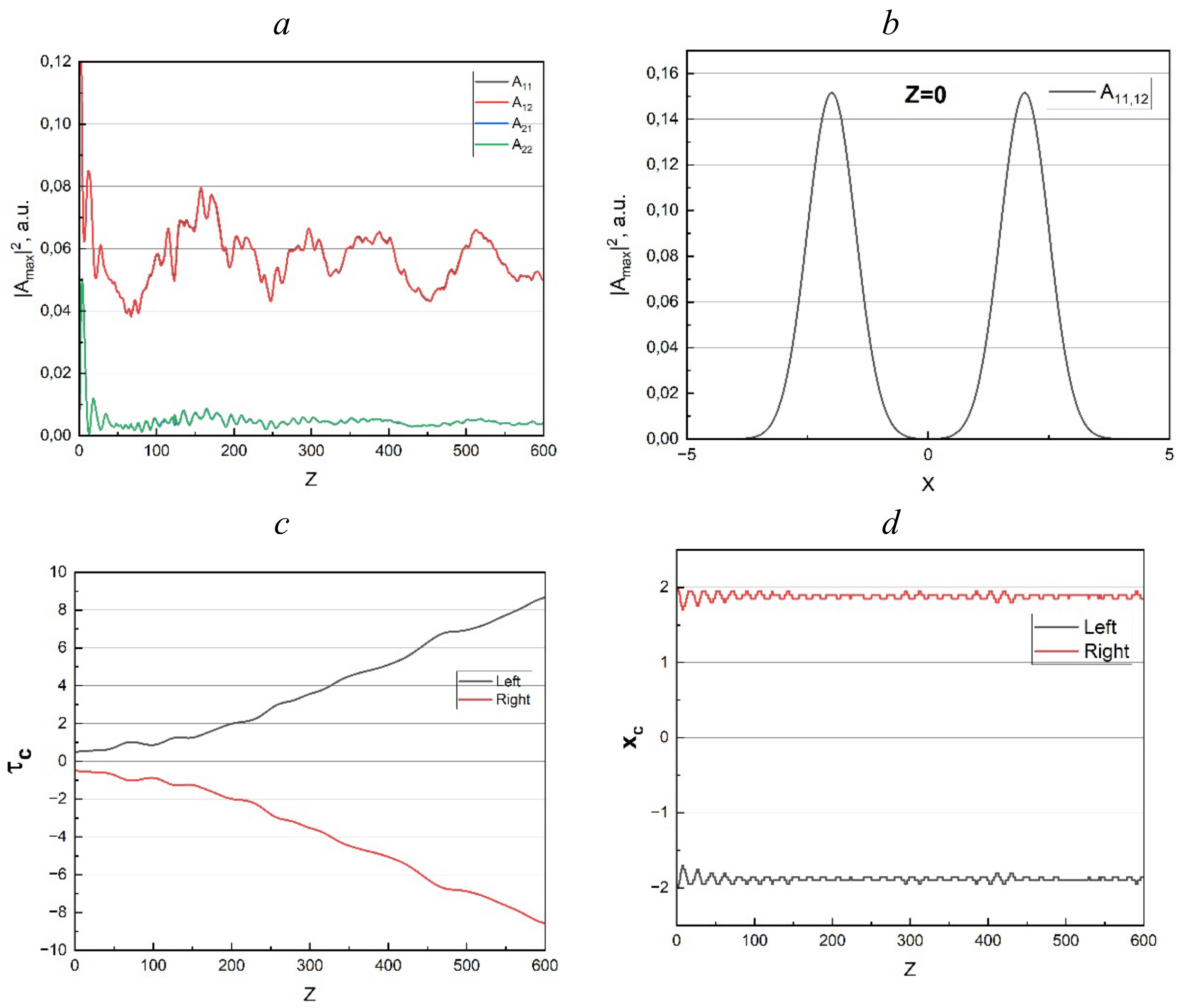
FIGURE CAPTIONS

Fig. 1. Generation of a pair of coupled solitons by nonsynchronous ($\tau_{10} = -0.5, \tau_{20} = 0.5$) in-phase ($\varphi_{11} = \varphi_{12} = 0$) beams of the fundamental frequency $E_{11} = 0.5, E_{12} = 0.5, E_{21} = 0, E_{22} = 0$. Peak intensities (a) of the fundamental frequency (black and red for the left and right waveguides, respectively) and the second harmonic (blue and green for the left and right waveguides, respectively). Transverse beam profiles at$\tau = 0$ of the fundamental frequency (red) and second harmonic (blue) for$z = 0$ (d),$z = 100$ (e),$z = 200$ (f). Dependence of the positions of spatial (b) and temporal (c) centers on the propagation distance. Waveguide parameters $x_w = 2, a_\omega = a_{2\omega} = 2, D_{q1} = 10, D_{q2} = 10$. Diffraction coefficients $D_{x1} = 0.1, D_{x2} = 0.05$, dispersion coefficients $D_{\tau1} = 0.05, D_{\tau2} = 0.1$, nonlinearity coefficient $\gamma = 0.5$.

Fig. 2. Generation of a pair of coupled solitons by nonsynchronous ($\tau_{10} = -0.5, \tau_{20} = 0.5$) beams of fundamental frequency $E_{11} = 0.5, E_{12} = 0.5, E_{21} = 0, E_{22} = 0$. Peak intensities (a) of the fundamental frequency in the right waveguide and the position of time centers of the pulse pair (b) as a function of distance z for different initial phases. Peak intensities (c) of the main frequency pulse pair and the position of their time centers (d) for the case $\varphi_{11} = 0, \varphi_{12} = 0.6\pi$. Parameters of the waveguide $a_\omega = a_{2\omega} = 2, D_{q1} = 10, D_{q2} = 10$. Diffraction coefficients $D_{x1} = 0.1, D_{x2} = 0.05$, dispersion coefficients $D_{\tau1} = 0.05, D_{\tau2} = 0.1$, nonlinearity coefficient $\gamma = 0.5$.

Fig. 3. Generation of a pair of coupled solitons by nonsynchronous ($\tau_{10} = -0.5, \tau_{20} = 0.5$) in-phase ($\varphi_{11} = \varphi_{12} = 0$) beams of fundamental frequency $E_{11} = 0.5, E_{12} = 0.5, E_{21} = 0, E_{22} = 0$. Peak intensities (a) of fundamental frequency and position of

spatial centers (b) as a function of distance z for $x_w = 1.8, 1.9, 2.0$. Transverse profiles of the beams at $\tau = 0$ and at $x_w = 1.9$ for the fundamental frequency between $z = 20$ and $z = 40$ with a step of $z = 5$ (c). Waveguide parameters $a_\omega = a_{2\omega} = 2, D_{q1} = 10, D_{q2} = 10.$ Diffraction coefficients $D_{x1} = 0.1, D_{x2} = 0.05$, dispersion coefficients $D_{\tau1} = 0.05, D_{\tau2} = 0.1$, nonlinearity coefficient $\gamma = 0.5$.



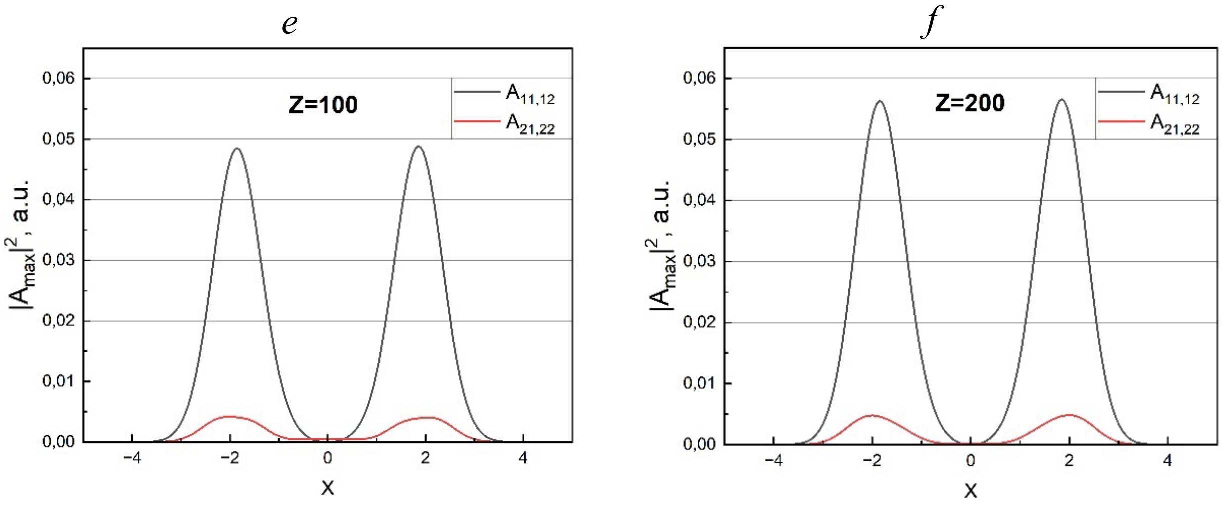
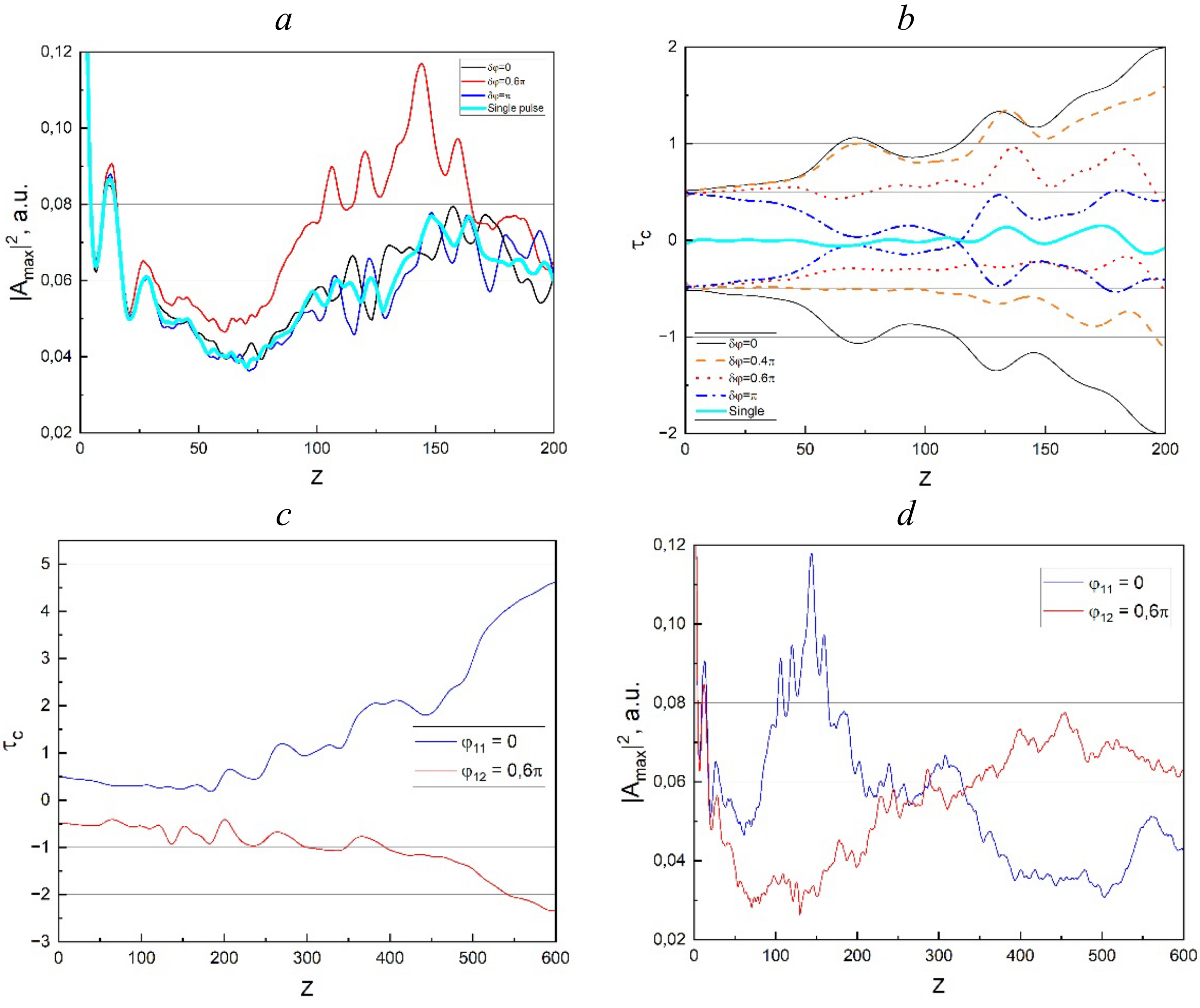

e
f
Z=100
Z=200
A11,12
A21,22
|Amax|2 , a.u.
x

Fig. 1


a
b
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d
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δφ=π
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z
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φ12 = 0,6π

Fig. 2

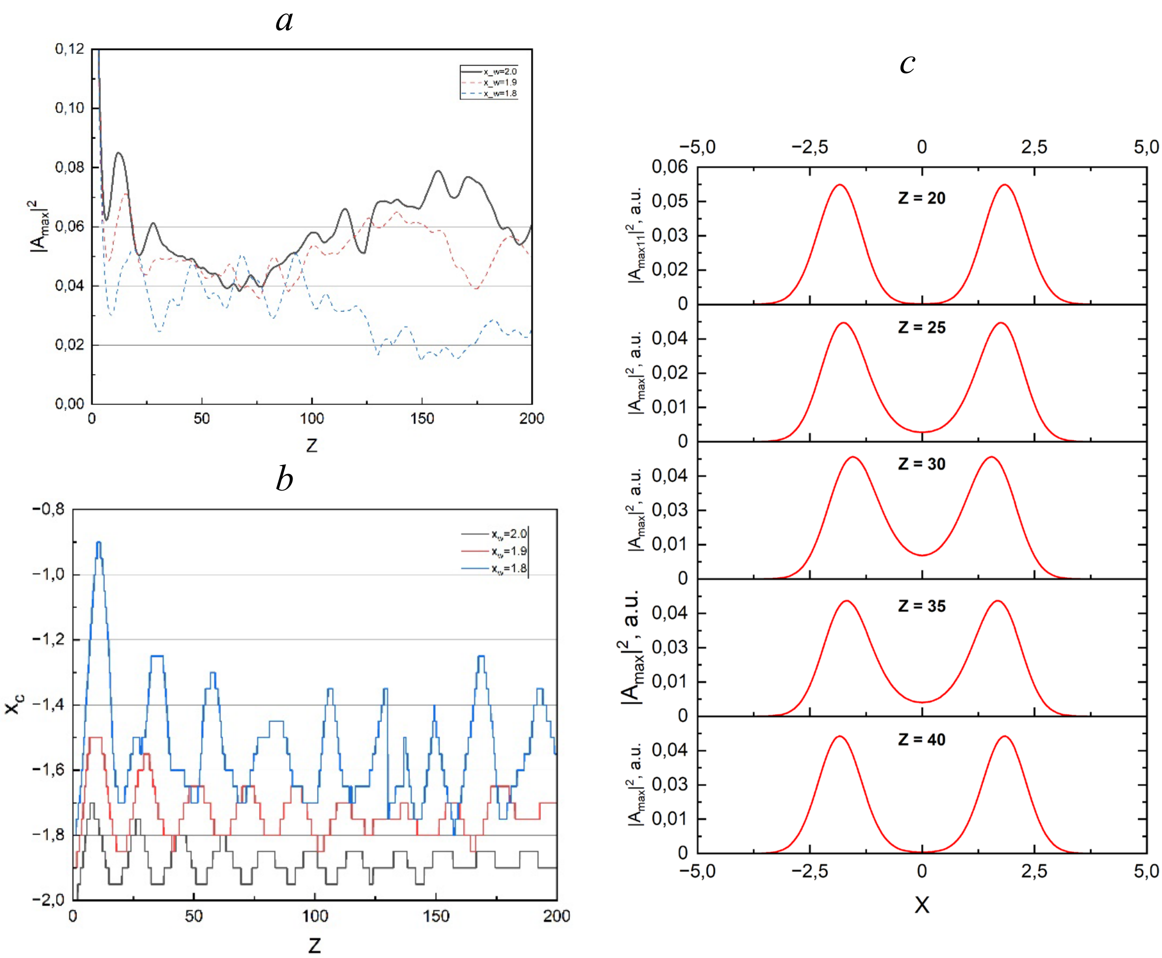

Fig. 3