PROPAGATION OF FRACTAL SPECKLES IN OPTICAL SYSTEMS AND IN FREE SPACE

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Received September 06, 2024

Revised September 16, 2024

Accepted September 30, 2024

Abstract. The diffraction transformation of wave fractal fields is considered. It is shown that when light beams with a speckle structure propagate in optical systems and in free space, their fractal properties have a high degree of stability.

Keywords: *speckles, fractal, optical system, Fourier transform*

DOI: 10.31857/S03676765250107e9

INTRODUCTION

Studies of the properties of fractal speckle fields carried out within the framework of fractal optics have made it possible to find solutions to a number of important fundamental questions. The notion of scaling (scale invariance) has been clarified with respect to speckle structures [1], the fractality of the location of dislocation formations has been estimated [2], and the features of the transition from Rayleigh statistics of intensity distribution to non-Rayleigh statistics have been considered [3]. The applied aspects of the performed studies turned out to be quite significant. First of all, biomedical applications should be referred to them. Thus, the fractal speckle technology allowed us to develop new diagnostic methods [4-6], to increase the information capacity of communication systems [7], made it possible to improve treatment methods in ophthalmology [8-11] and art therapy [12-15].

As a rule, in practice, a light beam with a speckle structure travels some distance from the initial plane, where the speckle structure is formed, to the plane where the transverse intensity distribution is fixed for the purpose of this or that application. However, there is no complete information in the literature on the nature and degree of transformation of amplitude-phase, scaling and statistical characteristics of radiation during its propagation. The purpose of this work is to evaluate the self-consistent changes of these characteristics depending on the initially set parameters. At the same time, special attention is paid to finding the degree of adequacy of the initial field to its image in the optical system.

PROBLEM STATEMENT AND METHOD OF SOLUTION

As a staging part of the problem, consider a speckle beam incident on a collecting lens with focal length f . Let us place the initial plane just behind the lens, where the radius of curvature R of the beam wavefront is equal to f . Consider how the beam structure will change during focusing and what is the degree of correlation between the field distribution in the initial plane and in the image plane located at a distance from the lens $2R$.

At numerical modeling of the structure of the fractal speckle field in the initial plane the Weierstrass function was used, which has the form

$$W_{x,y} = \sigma \sum_{v=0}^{V} \sum_{n=0}^{N} \left[b^{(D-2)n} \cos \left[2\pi s b^n \left[\left(x - \frac{K+1}{2} \right) \sin(\alpha v) + \right.\right.\right.$$
$$\left.\left.\left. + \left(y - \frac{K+1}{2} \right) \cos(\alpha v) \right] + \psi_n + \psi_v \right] \right] + A. \quad (1)$$

Here$W_{x,y}$ - amplitude of the radiation field; ,xy - discrete transverse coordinates ($0 \leqslant x, y \leqslant K$);$\sigma$ - standard deviation of the amplitude from the mean value;N - number of harmonics;V - number of azimuthal partial waves;n - harmonic number;v - index of the azimuthal component of the wave; α - elementary azimuthal rotation angle;b - scaling parameter;s - scaling parameter;D - fractal dimension of the Weierstrass function graph at one-dimensional representation; ,$\psi_n \psi_v$ - phases of field components;A - component with homogeneous distribution of field amplitude. At random values of phases ,ψ_n ψ_v , a speckle field was formed, the intensity distribution density in which obeyed the Rayleigh statistics.

In order to take into account the sphericity of the beam wavefront at the lens exit, function (1) was multiplied by the correction function

$$F_{x,y} = e^{\frac{i\left[\left[xu - \frac{(K+1)u}{2}\right]^2 + \left[yu - \frac{(K+1)u}{2}\right]^2\right]\pi}{\lambda R}}. \quad (2)$$

Here the parameter$u$ characterizes the used degree of transverse coordinate discretization,λ is the wavelength, R is the radius of curvature of the wavefront,$i = \sqrt{-1}$. In some cases, an additional correction function$T$, which plays the role of a "soft" aperture, was used to reduce the influence of edge effects

$$T_{x,y} = \xi e^{-\left[\left(xu - \frac{(K+1)u}{2}\right)^2 + \left(yu - \frac{(K+1)u}{2}\right)^2\right]^4}, \quad (3)$$

whereξ is a constant value.

The following numerical simulation results illustrating the speckle beam propagation are obtained for the following set of parameters:$K = 255,\ \alpha = 2\pi/48$, $V = 47,\ v = 0 \ldots V,\ N = 5,\ n = 0 \ldots N,\ \ \sigma = 3.3,\ s = 0.05,\ b = 2, A = 0$. The random phases ,ψ_n ψ_v were set using the relations

$$\psi_n = \frac{rnd(n)4\pi}{(n+1)},\ .\psi_v = \frac{rnd(v)4\pi}{(v+2)} \qquad (4)$$

For the sake of clarity, let us assume that$R = 1.5$ м , and the wavelength $\lambda = 0.5 \cdot 10^{-6}$ м . Let us also assume that the size of the working field, defined by$K$, in metric dimension is$a = 0.02$ м . In the same dimension, the distance between significant points of the working field is .$u = a/K = 7.812 \cdot 10^{-5}$ м

To estimate the characteristics of the light field at different distances behind the screen, the method of decomposition of the initial field by plane waves, which is the basis of Fourier optics, was used [16]. It is realized in several stages. First, using the fast Fourier transform procedure, the spatial complex spectrum of the radiation is determined$S = \mathrm{cfft}(W)$. Then, taking into account the raids of plane waves at different distances z, a new complex spectrum is determined Q

$$Q_{x,y} = S_{x,y} \cdot \exp[i2\pi z_T(c(x)^2 + c(y)^2)]. \qquad (5)$$

For further calculations it is advisable to give it a centrally symmetric character

$$H_{x,y} = \left|Q_{\mathrm{mod}\left(x+\frac{K+1}{2},K+1\right),\mathrm{mod}\left(y+\frac{K+1}{2},K+1\right)}\right|. \qquad (6)$$

In formula (5), the distances$z$ are expressed as fractions of the so-called Talbo length$T = 2a^2/\lambda$, i.e.,$z_T = z/T$. The auxiliary functions included in this formula$c(t)$ have the form

$$c(t) = \mathrm{mod}\left(t + \frac{K+1}{2}, K+1\right) + (K+1)/2. \qquad (7)$$

Finally, in the last step of the procedure, the distribution of the field$B_{x,y}$ at a distance is determined by means of the inverse Fourier transform z_T

$$B = i\text{cfft}(H). \quad (8)$$

CALCULATION RESULTS

The calculation showed that, in accordance with the wave optics concepts, the initial light beam is first focused up to the focal distance$z = R$, and then diverges to form an image of the initial distribution at a distance$z = 2R$. This kind of beam transformation is shown in Fig. 1. It shows the distributions$|W_{x,y}|$. Fig. *1a* shows the distribution of$|W_{x,y}|$ just behind the lens assuming that the light beam is confined by a square aperture whose size is 3.2 times smaller than the working field size. The gradual reduction of the beam size during its focusing is illustrated in Fig. *1b*, where the light field is given at a distance$z = R/2$. The qualitative transformation of the beam structure occurs in the focal plane when$z = R$ (Fig. *1c*). According to the position of the Fourier optics in this plane, the field is the result of the Fourier transform of the initial distribution of the amplitude of the light oscillations. The Fourier image formed in the focal plane had the form of a system of concentric circles, which corresponded to the spatial frequency distribution of the fractal speckle beam. The presence of scaling in the Fourier image proves that the ratio of the radii of the circles was a constant value equal to the parameter$b = 2$ present in formula (1). A change in this parameter, which is essentially a scaling coefficient, led to a change in the radius ratio. The peculiarity of the spatial spectrum, caused by the presence of scaling, largely determines the effectiveness of the visual impact of fractal structures in carrying out therapeutic procedures in art therapy and ophthalmology. The matter is that in the cerebral cortex the processing of visual signals carrying information about images is carried out on the basis of the structure of their spatial spectra [14]. Due to the presence of scaling, there is no need to process spatial spectra in a wide frequency range, it is enough to fix only their low-frequency part. This accelerates and facilitates the process of visual perception of the objects under consideration and, as a consequence, creates a feeling of comfort and aesthetic pleasure. The resulting strengthening of connections between neurons in the cerebral cortex helps to cure a number of eye diseases (e.g. glaucoma). Propagating further from the focal plane, the beam increases in size and forms an image of the initial distribution at a distance$z = R$ (Fig. *1g*).

It was found that in the process of speckle beam propagation it retained fractal features regardless of statistically independent realizations of their structure. The fractal dimensions of the initial distribution and its image estimated by the covering method [17] turned out to be close to each other and amounted to the value of 2.5 ∓ 0.1. The minimum fractal dimension equal to 2.25 ∓ 0.05 corresponded to the field distribution in the focal plane. In parallel with the estimation of fractal dimensionality values at different distances from the initial plane, average speckle values were determined. This was done by a cutoff of 0.5 from the maximum value of the calculated autocorrelation function. The calculation showed that the sizes of speckles in the image due to diffraction broadening exceed their initial sizes by about 20%. A significant decrease in speckle size (2.5 times) was observed near the focal plane.

Calculation of the correlation coefficientη of the field distributions in the initial plane and in the image plane (Figs. *1a* and *1g*) gave the value$\eta = 0.53$. Increasing the fractal dimension D led to a decrease η . This can be clearly seen from the course of the curve shown in Fig. 2. In the most important region for practical applications ($D < 1.5$) the decrease of the value of η relative to the maximum value does not exceed 30%.

DIFFERENT WAVEFRONT STRUCTURE

In order to take into account the influence of spherical aberration in the initial plane on the radiation characteristics, formula (2) was given a different form

$$F_{x,y} = e^{i\left[\left(\frac{2x}{K+1}-1\right)^2+\left(\frac{2y}{K+1}-1\right)^2\cdot\left[1+\rho\left[\left(\frac{2x}{K+1}-1\right)^2+\left(\frac{2y}{K+1}-1\right)^2\right]\right]\right]\cdot\frac{\pi\left(\frac{a}{2}\right)^2}{\lambda R}}. \quad (9)$$

The parameter ρ included in expression (9) characterizes the degree of influence of aberration on the image structure. In Fig. 3, this influence is graphically represented as a change in the correlation coefficient η of the image and the initial field distribution. The figure shows that the decrease of the correlation coefficient exceeding 10% occurs at .$|\rho| > 0.1$

The possibility of using the developed software for the case of speckle wave propagation with an initially flat wavefront was also considered. The plane wave method used therein required some adjustments compared to the previous case. This is due to the fact that the divergence of the speckle beam requires an increase in the working field size due to the need to take into account the peculiarities of the beam structure at its periphery. The mentioned problem was overcome by using an adaptive scheme of permanent increase of the working field size. It was shown that at a distance from the initial plane $z_1 = 0.0001\, d^2/\lambda$, where d is the working field size, the intensity distribution retains the properties characteristic of a speckle fractal beam. Quantitative analysis of the transformation of the speckle wave structure allowed us to establish that such field characteristics as probability density and radius of correlation of intensity values, their standard deviation in the region $0 < z < z_1$, depending on the realization, can undergo noticeable and sometimes significant changes. At the same time, the fractal dimension calculated by the covering method experienced deviations from the mean value equal to 2.45 not exceeding 2%. This indicates the stability of such an important characteristic of the speckle field as its fractal dimension.

CONCLUSION

Propagation in optical systems and in free space of fractal speckle beams of Rayleigh type is characterized by a number of important physical regularities. In addition to the fact that during propagation the scaling structure of their spatial spectrum remains unchanged, both the fractal shape and the probability density of the transverse intensity distribution are preserved. However, the value of the fractal dimension, fixed at different distances, can generally vary within large limits. At the same time, the fractal dimensions of the beams in the initial plane and in the image plane are close to each other. The correlation coefficient of the initial field and its image depends on the fractal dimension set in the initial plane and decreases with increase. Thus, increasing the fractal dimension from the value of 2.25 to 2.7 can reduce the correlation coefficient by a factor of 2. The correlation coefficient is noticeably affected by the presence of spherical aberration in the beam formed in the initial plane. If the aberration contribution to the wavefront structure exceeds 10%, a sharp drop in the correlation coefficient should be considered. Additional analysis of the process of propagation of a fractal speckle beam in free space has shown that in this case, too, it retains fractal properties.

ACKNOWLEDGEMENT

The authors express their gratitude to Prof. A.N. Bogolyubov of the Physics Department of Moscow State University for support of this research direction.

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FIGURE CAPTIONS

Fig. 1. Variation of the beam structure in the optical system. z = 0 (a), R/2 (b), *R* (c), *2R* (d).

Fig. 2. Effect of fractal dimension *D* on the value of correlation coefficient η.

Fig. 3. Effect of spherical aberration on image structure.

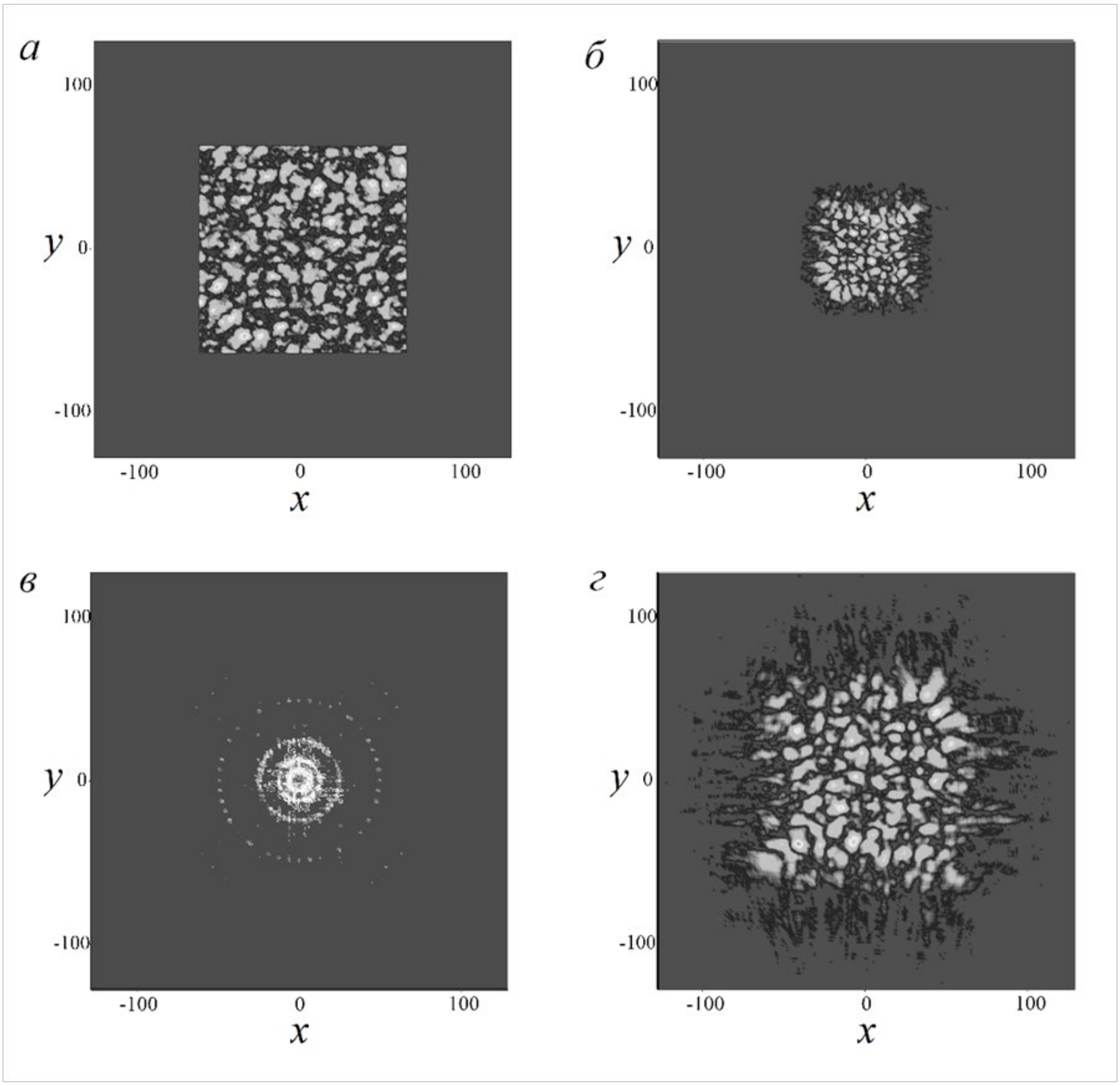
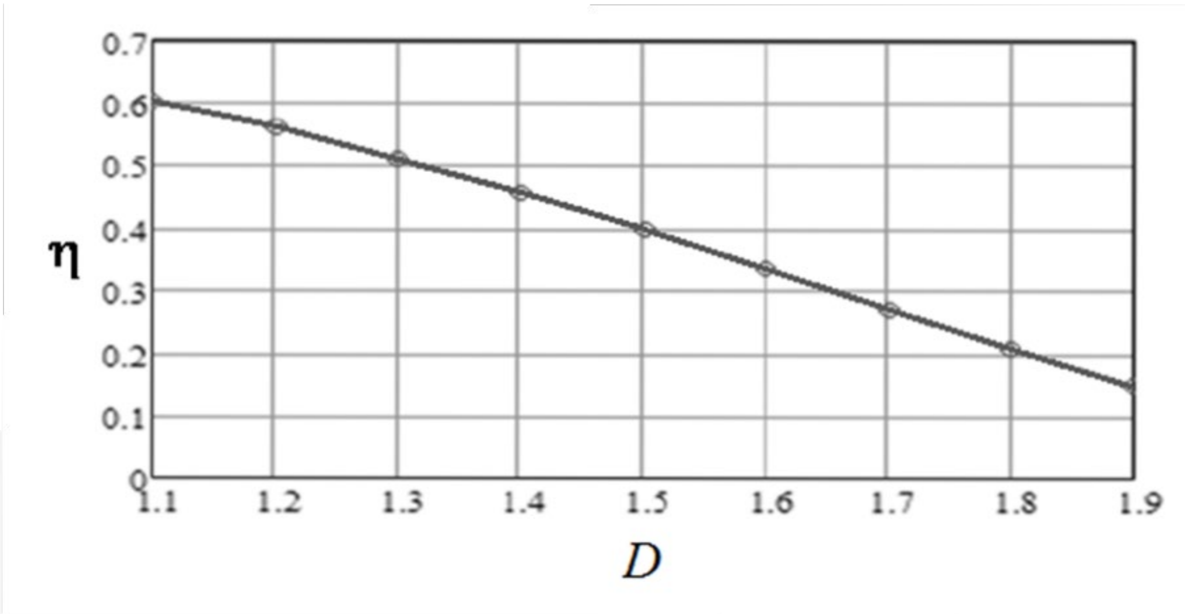


Fig.1


0.7
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D

Fig. 2

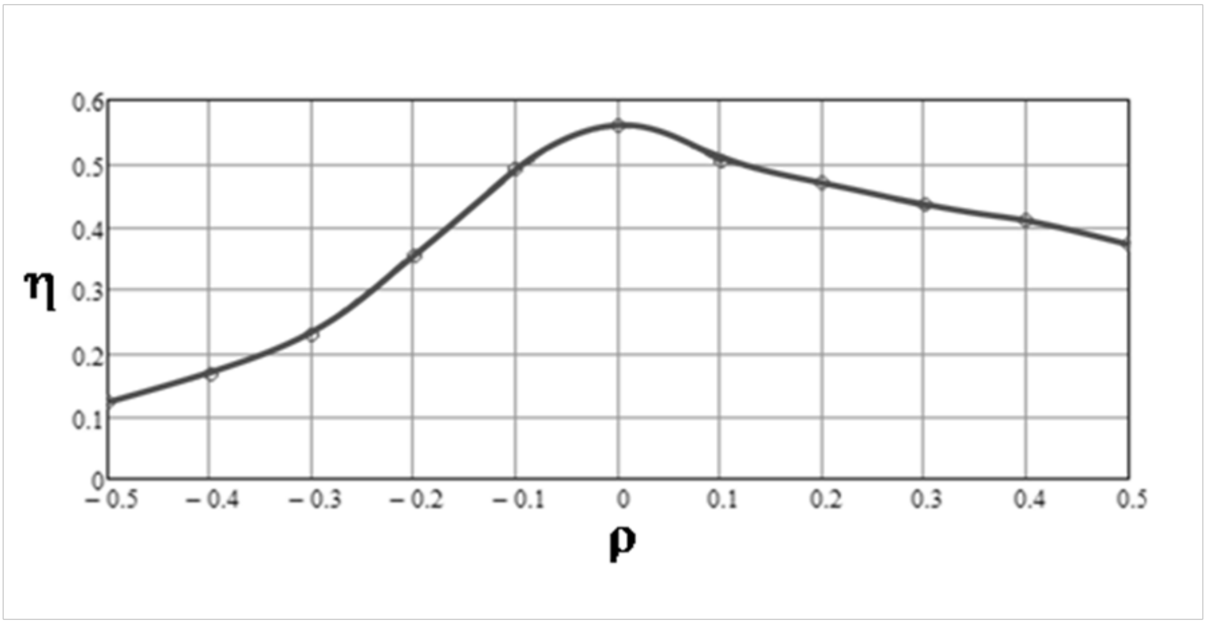
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Fig. 3