RADIOELECTRIC EFFECT IN A SUPERLATTICE BASED ON A 3D DIRAC CRYSTAL

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Abstract. A kinetic theory for the radioelectric effect in a superlattice based on a 3D Dirac crystal in a constant electric field has been constructed. The current density has been shown to get the resonance in the case where the Bloch frequency is a multiple of the frequency of the electromagnetic wave. The latter can lead to a change in the direction of the current density. The amplitude dependence of the radioelectric current density has been studied.

Keywords: *radioelectric effect, superlattice, Dirac crystal, Bloch frequency*

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INTRODUCTION

Currently, the electronic properties of three-dimensional (3D) structures based on Dirac or graphene-like crystals [1,2], which have a wide range of applications, are being actively studied [2-5]. On the one hand, the charge carriers of 3D Dirac materials are characterized by the presence of three degrees of freedom of motion and, on the other hand, by the relativistic form of the dispersion law [6,7]. There are various ways to fabricate such structures [8,9]. In [10], a 3D structure is proposed, which is a superlattice (SL) with graphene sheets separated by semiconductor spacers along the growth axis. In [11], a heterostructure consisting of periodically alternating layers of a topological insulator and a dielectric playing the role of a quantum barrier was considered. Attention to SRs based on Dirac crystals is due, in particular, to the possibility of using the latter as a working medium for the generation of solitary electromagnetic (EM) waves of a new type predicted in [12] and recently attracting the attention of researchers, including those abroad [13-15].

The so-called radioelectric effect, which consists in the entrainment of free charge carriers by EM radiation in the direction of its propagation [16], can be taken as a basis for the methods of diagnostics of semiconductor structures, as well as detection of EM waves propagating in them. This effect with respect to standard 3D semiconductor SRs has been theoretically studied in [16,17] for different wave polarizations. The entrainment of conduction electrons by a solitary wave in a graphene SR was considered in [18]. However, the observation of this phenomenon in the latter case requires a sufficiently precise orientation of the wave polarization plane along the graphene plane, which is a challenging experimental task. In addition, the creation of a free graphene sheet is hampered by the inevitable occurrence of surface defects and deformations [19], in order to eliminate which special substrates are used. It is easy to see that 3D materials are devoid of such necessity.

In the present work, a kinetic theory of the radioelectric effect in SRs based on a 3D Dirac crystal, as well as the influence of this effect on the longitudinal voltammetric characteristic (VAC) of the considered structure, is constructed. Note that the WAC of SRs in the EM absorption regime was studied in [20], where the emerging areas of WAC with absolute negative conductivity (AOP) were interpreted as a result of resonant absorption by electrons of EM field quanta and optical phonons, leading to the corresponding quantum transitions along the Starkov ladder. Below is shown the possibility of AOP in a different situation, and namely in the mode of entrainment of charge carriers by an EM wave polarized along a circle. Moreover, in contrast to [20] here the conditions for the Starkov quantization will not be required, and the AOP sites can be described in the framework of the quasi-classical approach based on the relaxation time approximation.

ELECTRON SPECTRUM OF SR

The SR considered below is a multilayer heterostructure consisting of alternating layers of a 3D Dirac crystal and a conventional insulator acting as a spacer material (Fig. 1). At this point, such a structure may well be fabricated using available techniques [21,22]. The Hamiltonian describing this structure can be written as follows: $\hat{H}_{\mathrm{SL}} = \upsilon_{\mathrm{F}}\tau_x \otimes \vec{\sigma}\cdot\vec{p} + \tau_z V(z)$. Here, the Pauli matrices $\vec{\sigma}$ and $\vec{\tau}$ are responsible for the spin and pseudospin degrees of freedom, respectively [7], $\otimes$ is the Kronecker product operation, υ_{F} is the velocity on the Fermi surface, $\vec{p}$ is the three-dimensional momentum operator, $V(z)=V(z+d)$ is an additional scalar potential due to the alternation of quantum wells and barriers along the *Oz* axis, and *d* is the SR period. In the low-energy one-minizone approximation, the dispersion law for electrons in the conduction minizone can be written in the following form ($\hbar = 1$):

$$\varepsilon(\vec{p}) = \varepsilon_\perp(p_\perp) + \Delta(1 - \cos p_z d), \tag{1}$$

where $\vec{p}_\perp$ is the electron momentum component transverse to the CP axis, , $\varepsilon_\perp(\vec{p}_\perp) = \sqrt{\Delta_g^2 + \upsilon_F^2 p_\perp^2}$ Δ_g is the half-width of the energy gap between the conduction zone and the valence zone, Δ is a structural parameter expressed through integrals of the wave function overlap from neighboring quantum wells and has the meaning of the half-width of the conduction minizone. It is considered that the inequality is fulfilled: .$\Delta << \Delta_g$

INFLUENCE OF THE ENTRAINMENT EFFECT ON THE LONGITUDINAL SAC

Let us place the above considered SR in the field of an EM wave polarized along a circle and propagating against the *Oz* axis as shown in Fig. 1. The projections of the EM wave field intensity vectors on the coordinate axes are as follows:

$$E_x = E_0 \cos(\omega t + qz), , , , E_y = E_0 \sin(\omega t + qz)\, H_x = E_y\, H_y = -E_x \tag{2}$$

where E_0, q and ω are the amplitude, wave number and frequency of the wave, respectively. We consider that the structure has electronic conductivity. According to the chosen orientation of the intensity vectors, the wave field transfers momentum to the electrons, entraining them against the *Oz* axis. Consequently, the entrainment current density in the absence of a constant field is directed along the *Oz* axis. Let us calculate the longitudinal to the CP axis component of the current density j_z under the condition of simultaneous action of both the wave field and the constant electric field with the strength $\vec{E}^{\mathrm{dc}}$, directed along the *Oz* axis. For this purpose, let us use the following formula:

$$j_z = -e\sum_{\vec{p}} \upsilon_z(\vec{p}) f(\vec{p},t), \tag{3}$$

where $,\vec{\upsilon} = \partial_{\vec{p}}\varepsilon \; f(\vec{p},t)$ is the nonequilibrium distribution function that takes into account the action of force fields and is a solution of the kinetic Boltzmann equation:

$$\frac{\partial f}{\partial t} - e\left(\vec{E}^{\mathrm{dc}} + \vec{E} + \frac{1}{c}[\vec{\upsilon}, \vec{H}\,] \right)\cdot\frac{\partial f}{\partial \vec{p}} = -\frac{f(\vec{p},t) - f_0(\vec{p})}{\tau}. \tag{4}$$

Here $f_0(\vec{p})$ is the equilibrium distribution function, τ is the relaxation time. As in [16,17], it is assumed that the EM wavelength significantly exceeds the free path length of charge carriers. Therefore, in (4) the summand with the spatial derivative $\partial_{\vec{r}} f$ is omitted. The latter also allows us to neglect the coordinate dependence of the wave field strengths (2). Let us introduce the notation: $\gamma = d\Delta_g c^{-1}$. The parameter γ is of order $\gamma \sim \upsilon c^{-1}$, where υ is the characteristic velocity of charge carriers, and $\upsilon << c$. In the zero approximation for the small parameter υc^{-1} we can neglect the action of the magnetic field so that the solution (4) has the form:

$$f^{(0)}(\vec{p},t) = \frac{1}{\tau}\int_{-\infty}^{t} \mathrm{d}t_1 e^{-\frac{t-t_1}{\tau}} f_0\left(\vec{p} + \frac{e}{c}\left(\vec{A}_1 - \vec{A}\right)\right), \tag{5}$$

where $\vec{A}(t)$ is the vector potential of the EM field, $\vec{A}_1 = \vec{A}(t_1)$. Let us substitute (5) into (3) and take into account the parity of the equilibrium distribution function. As a result, we arrive at the classical expression describing the longitudinal VAC of SR [23]:

$j_z^{(0)} = j_0 \Omega_{\rm B} \tau \left(1 + \Omega_{\rm B}^2 \tau^2\right)^{-1}$. Here denoted by: $j_0 = n_0 e d \Delta , \Omega_{\rm B} = e E_z^{\rm dc} d$ is the Bloch frequency, n_0 is the concentration of free charge carriers in the conduction band. The correction for the distribution function in the following approximation has the form:

$$f^{(1)}(\vec{p},t) = \frac{e}{c} \int_{-\infty}^{t} {\rm d}t_2 e^{-\frac{t-t_1}{\tau}} \left[\vec{\upsilon}\left(\vec{p} + \frac{e}{c}\left(\vec{A}_1 - \vec{A}\right)\right), \vec{H}(t_1) \right] \cdot \frac{\partial}{\partial \vec{p}} f_0 \left(\vec{p} + \frac{e}{c}\left(\vec{A}_1 - \vec{A}\right)\right). \quad (6)$$

After substituting (6) into (3) and some transformations, we arrive at the following expression for the correction to the current density:

$$j_z^{(1)} = \frac{e}{c} \int_0^{\infty} e^{-\xi} \cos(\Omega_{\rm B} \tau \xi) {\rm d}\xi \cdot$$

$$\cdot \sum_{\vec{p}} \left(-\frac{\partial f_0}{\partial \varepsilon} \right) \left(\varepsilon_\perp \left(\vec{p}_\perp + \frac{e}{c} \vec{A}(t - \tau\xi) - \frac{e}{c} \vec{A}(t) \right) - \varepsilon_\perp (\vec{p}_\perp) \right) \upsilon_z^2 (p_z). \quad (7)$$

Further calculations will be carried out for the case of extremely low temperatures such that we can make a substitution: $(-\partial_\varepsilon f_0) \rightarrow \delta(\varepsilon - \varepsilon_{\rm F})$, where $\delta(\xi)$ is the delta function, $\varepsilon_{\rm F}$ is the energy corresponding to the Fermi level. We consider that the latter is located inside the conduction band near its bottom so that the inequality is satisfied: $0 < \varepsilon_{\rm F} - \Delta_{\rm g} << \Delta_{\rm g}$. If we take the following characteristic for Dirac and graphene-like materials numerical value of the energy gap $\Delta_{\rm g}\sim$ 50 meV, then the concentration of free electrons in the conduction band $n_0\sim 10^{14}{\rm cm}^{-3}$ corresponds to the Fermi energy equal to $\varepsilon_{\rm F}\sim$ 51 meV, which quite satisfies the above inequality. According to the latter, only those electrons near the bottom of the minizone, i.e., electrons with small momentum values, will participate in electron transfer at low temperatures: $p_\perp \upsilon_{\rm F} << \Delta_{\rm g}$, $p_\perp \omega << e E_0$. As a result of the calculations in (7), performed taking into account the above conditions, we arrive at the following result:

$$\frac{j_z^{(1)}}{j_0}=\frac{\gamma\Delta}{\varepsilon_F-\Delta_g}\left(\frac{S_0}{1+\Omega_B^2\tau^2}+\sum_{n=1}^{\infty}S_n\left(\frac{1}{1+(\Omega_B-n\omega)^2\tau^2}+\frac{1}{1+(\Omega_B+n\omega)^2\tau^2}\right)\right), \quad (8)$$

where denoted

$$S_n(a_0)=\frac{2}{\pi}\int_0^{\pi/2}\sqrt{1+a_0^2\sin^2\xi}\cos(2n\xi)d\xi\ , \quad S_0(a_0)=\frac{2}{\pi}E(-a_0^2)-1\ , \quad (9)$$

$E(\xi)$ - is the full elliptic integral of the 2nd kind, $a_0=2\omega^{-1}\Delta_g^{-1}\upsilon_F eE_0$ is the dimensionless amplitude of the electric field strength of the wave. The dependence of the resulting current density, equal to $j_z=j_z^{(0)}+j_z^{(1)}$, on the intensity E_z^{dc} is shown in Fig. 2 by a solid line. Here also the dashed line shows the longitudinal VAH of SR in the absence of EM wave. It can be seen that the areas of negative differential conductivity in both cases practically coincide except for resonance situations when the Bloch frequency is a multiple of the EM wave frequency: $\Omega_B=n\omega$. In the latter case, the current density goes sharply into the region of negative values, which corresponds to the AOP. A similar situation occurred for SRs with a parabolic transverse spectrum [17], where the entrainment current was also calculated in a linear approximation using the parameter $\upsilon c^{-1}\sim\gamma$. However, in [17] the resonance corresponding to AOP appeared only in one case: $\Omega_B=\omega$. The appearance of other AOP regions was a higher order effect of γ in [17] and required sufficiently large wave intensities, at which the WAC was significantly distorted compared to the WAC in the absence of the entrainment effect. According to (8), in the case of SRs based on Dirac crystals, already in the first approximation in γ there is a series of resonances, which is a consequence of the nonparabolicity of the transverse spectrum of the SR considered here.

Let us calculate the current density in the *k-th* resonance. For this purpose, let's put $\Omega_{\mathrm{B}} = k\omega$ in (8) and leave only the summand with $n = k$ due to the inequality $\omega\tau >> 1$. As a result we have: $j_z^{\mathrm{res}} \sim -\gamma j_0 |S_k(a_0)|$. The plots of the dependence of the current density on the wave amplitude in the first two resonances are shown in Fig. 3. It is easy to show that in the case of small wave amplitudes ($a_0 << 1$) the current density in the *k-th* resonance is proportional to a_0^{2k} . In the case of large amplitudes ($a_0 >> 1$), on the other hand, the resonance value is linear in amplitude. The latter result distinguishes the effect considered here from the analogous one in [17], where in the linear in γ approximation the resonance value is quadratic in the amplitude of the wave. The linear in amplitude current density at $a_0 >> 1$ is a direct consequence of the relativistic character of the transverse spectrum of charge carriers.

ENTRAINMENT EFFECT IN THE ABSENCE OF A CONSTANT FIELD

In the absence of a constant field, the electric current along the CP axis exists only due to the entrainment of electrons by the EM wave. Since the momentum of the EM wave imparted to the conduction electrons is directed against the *Oz* axis, the corresponding current density has a positive projection j_z . Substituting into (8) $\vec{E}^{\mathrm{dc}} = 0$, and considering the inequality $\omega\tau >> 1$, we write:

$$j_z = \frac{\gamma j_0 \Delta S_0(a_0)}{\varepsilon_{\mathrm{F}} - \Delta_{\mathrm{g}}}. \tag{10}$$

The graph of the dependence of the entrainment current density on the dimensionless wave amplitude a_0, plotted by formula (10), is shown in Fig. 3 with a dashed line. As expected, the plot for the current density at $\vec{E}^{\mathrm{dc}} = 0$ lies in the positive region. It follows from (10) that for small amplitudes, the entrainment current density is quadratic in amplitude: $j_z \sim \gamma\, j_0 a_0^2$, and for large amplitudes it is linear: $j_z \sim \gamma\, j_0 a_0$. Note that the amplitude dependence of the radielectric current is similar to the corresponding dependence for the EM radiation power absorbed by the graphene-like material [24]. This feature can serve as an additional indication of the correctness of the calculation results. Indeed, due to the absorption of the radiation energy by charge carriers, the impulse of the EM wave field is transmitted to electrons, which ensures the entrainment of the latter along the radiation propagation direction.

CONCLUSION

As a result of the study of the entrainment effect in SRs based on Dirac crystals, the following results were obtained. First, the effect of electron entrainment by the EM wave modifies the longitudinal SAH of SRs in such a way that a series of resonances appears: the current density experiences a sharp change whenever when the Bloch frequency is a multiple of the wave frequency ($\Omega_{\mathrm{B}} = n\omega$). Attention should be paid to the following peculiarity in the behavior of the entrainment current density when a constant electric field with resonant strength is switched on. As shown above, despite the positive value of the DC field strength ($E_z^{\mathrm{dc}} > 0$), the projection of the current density j_z in resonance, according to the plots in Fig. 3, does not increase, but, on the contrary, decreases up to the change of its sign. Moreover, this effect is more noticeable the higher the intensity of EM radiation.

Second, each of such resonances is a first-order effect in terms of the small parameter γ in contrast to [17], where only one resonance appeared for SRs based on materials with a quadratic law of dispersion of their carriers in the corresponding approximation. Third, for large wave amplitudes, the entrainment current density grows linearly with amplitude, which is a direct consequence of the relativistic character of the transverse spectrum of the SR considered here. The latter can be used as a basis for methods of laboratory diagnostics of the transverse spectrum of charge carriers of SRs based on Dirac crystals.

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CONFLICT OF INTERESTS

The authors of this paper declare that they have no conflicts of interest.

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FIGURE CAPTIONS

Fig. 1. Schematic of SR and configuration of EM fields: 1) 3D Dirac crystal layer, 2) insulator spacer layer, 3) EM wave polarized in a circle.

Fig. 2. Longitudinal VAC of SR modified due to the radioelectric effect (solid line, $\omega\tau = 30$, $a_0 = 20$) and VAC of SR in the absence of EM wave (dashed line).

Fig. 3. Dependence of the radioelectric current density on the dimensionless wave amplitude a_0: $\omega\tau = 30$, 1) $\Omega_B = \omega$, 2) $\Omega_B = 2\omega$, 3) $E^{dc} = 0$.

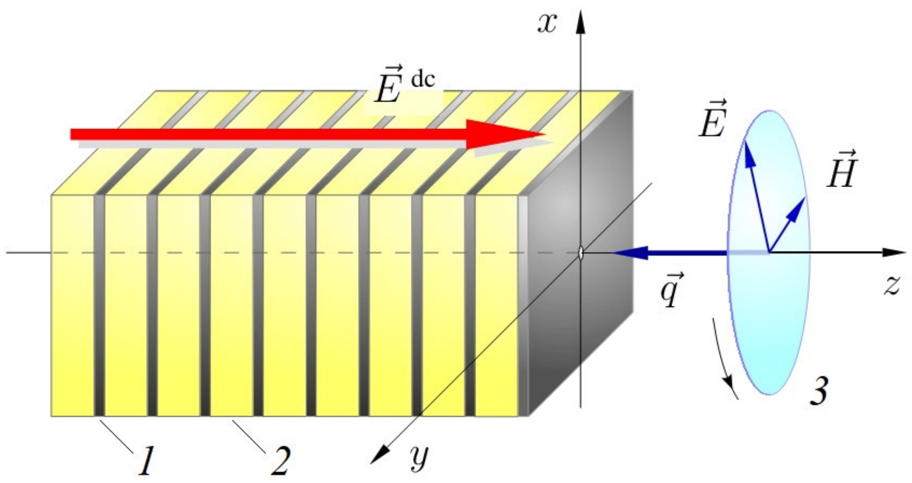

Fig 1.

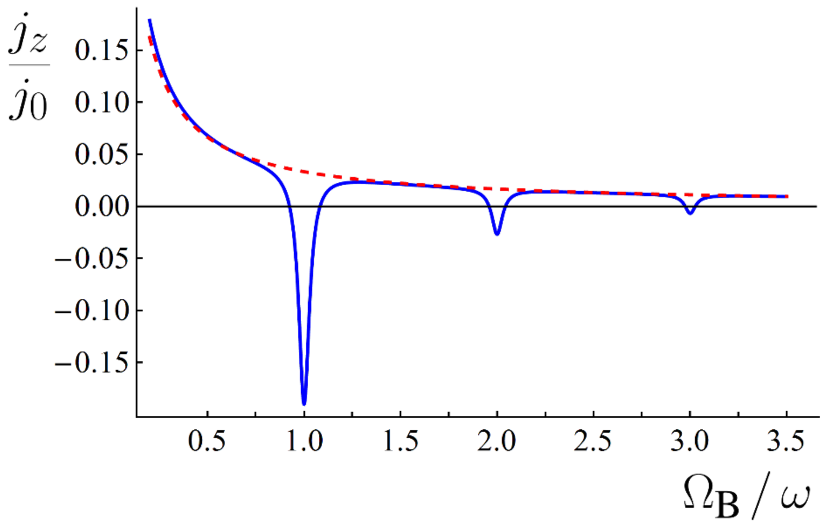

Fig. 2.

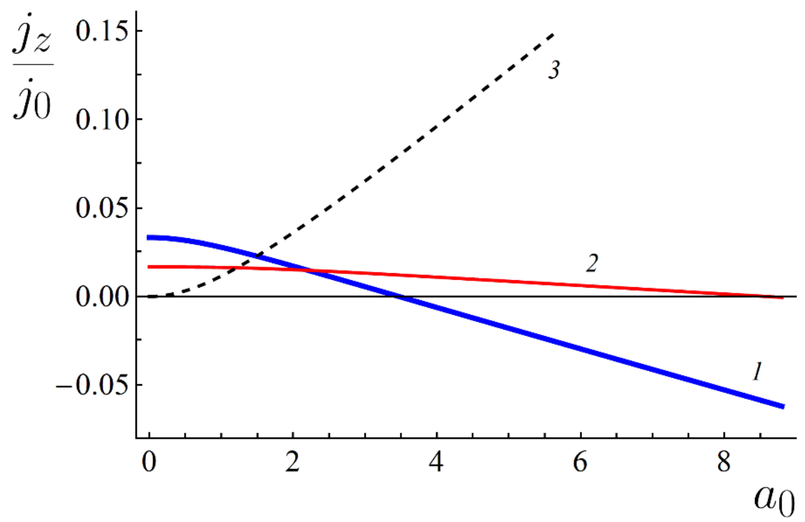

Fig. 3.