# OPTICAL PULSES IN A NON-HERMITIAN MEDIUM NEAR A SINGULARITY

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**Abstract.** The spectral method was used to solve the problem of interaction of short optical pulses with RT-symmetric photonic crystals under conditions of frequency singularity. It is shown that with a small deviation from the singular point of spontaneous decay of PT-symmetric field modes, a frequency singularity of the transmission and reflection coefficients of the structure arises. This leads to a significant narrowing of the pulse spectra and an increase in their amplitude and duration with unidirectional Bragg reflection.

**Keywords:** *RT-symmetry, spectral singularity, short optical pulses, dynamic Bragg diffraction*

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## INTRODUCTION

After states with real eigenvalues of energy [1, 2] were found in non-hermitian quantum-mechanical systems with parity-time (parity-time, PT-symmetry), this idea was generalized to various physical wave processes - optical [3-6], acoustic [7], in coupled mechanical oscillators [8], in electric circuits [9] and others. In optics, in non-hermitian media with PT-symmetric complex function of dielectric permittivity $\varepsilon(\vec{r}) = \varepsilon^*(-\vec{r})$ , i.e., in media with amplification and absorption (Fig. *1a*), PT-symmetric field modes with real values of wave numbers can propagate [3, 4]. Examples of periodic media of this type are described in detail, for example, in [10-13]. An important feature of PT-symmetric media is the presence of a special point (SP) of spontaneous decay of PT-symmetric states [14, 15], in which the degeneracy of eigenwaves occurs and PT-unsymmetric modes propagating with amplification and absorption appear when the balance of amplification and absorption changes. In the vicinity of the OT, new optical phenomena are observed, such as: unidirectional Bragg reflection, or unidirectional invisibility [16-18]; increased transparency of passive PT-symmetric media with an increase in their absorption capacity [14, 19]; asymmetric fission of short laser pulses at a special point in a dispersing RT-symmetric medium [20]; changes in the radiation structure of high-power diode lasers at the appearance of RT-symmetric modes in a laser resonator [21]; frequency singularity [22, 23]. However, the interaction of monochromatic waves or extended nanosecond pulses with PT-symmetric media has been considered so far. The propagation of short picosecond and subpicosecond pulses, especially near the frequency singularity, has remained poorly studied.

This paper studies the interaction of short optical pulses with PT-symmetric periodic structures, or photonic crystals (PCs), in the case of frequency singularity of spectral reflection coefficients $R(\omega)$ and passage $T(\omega)$ , including material dispersion. The problem of linear dynamic Bragg pulse diffraction in FCs is solved by the spectral method in the two-wave approximation. The method of broadband quasi-RT symmetry

[24] was used to recover the PT-symmetric properties of the medium. It is shown that at the special point of decay of RT-symmetric field modes, the reflection and transmission coefficients of photonic crystals of finite thickness do not have singularities, and the durations of reflected and transmitted pulses change insignificantly. However, even at a small deviation from the OT at a certain thickness of the FC, frequency singularities of the coefficients $R(\omega)$ and $T(\omega)$ appear. As a result, there is a significant narrowing of the spectra of pulses and an increase in their duration. There is also unidirectional Bragg reflection and amplification of pulses in the case of broadband quasi-RT symmetry in the FC with material dispersion.

## THEORY

Let a short optical pulse (wave packet) fall on the surface of $z=0$ bounded one-dimensional resonant PT-symmetric FC (Fig. 1)

$$E_{in}(\vec{r},t) = A_{in}(x,t)\exp(i\vec{k}_0\cdot\vec{r} - i\omega_0 t), \quad (1)$$

where $A_{in}(x,t)$ is the complex slowly varying amplitude, $,\vec{k}_0 = (k_0\sin\theta, sk_0\cos\theta)$ $\theta$ is the angle of incidence, $k_0 = \omega_0/c$, $\omega_0$ is the center frequency, $c$ is the speed of light, $s=+1$ when falling on the left, $s=-1$ when falling on the right (Fig. *1b*).

The dielectric permittivity of a PT-symmetric PC is described by a function of the form

$$\varepsilon(z,\omega) = \varepsilon_0 + \varepsilon'\cos(hz) + \tilde{\varepsilon}'(\omega)\sin(hz) + i\tilde{\varepsilon}''(\omega)\sin(hz), \quad (2)$$

where $\varepsilon_0 + \varepsilon'\cos(hz)$ is a real even function, $\tilde{\varepsilon}'(\omega)\sin(hz)$ and $\tilde{\varepsilon}''(\omega)\sin(hz)$ are odd functions of the real and imaginary parts of the dielectric permittivity due to the resonant atoms, $h = 2\pi / d$ is the modulus of the inverse lattice vector of the FC, $d$ is the lattice period. The appearance of the odd real summand in (2) is related to the Kramers-Kronig relation, which follows from the principle of causality and inevitably leads to the violation of the resonant medium RT-symmetry condition for the part of the spectrum of the optical pulse [25]. However, the use of the broadband quasi-RT-symmetry condition, when the pulse spectrum width is much smaller than the spectral line width of the inhomogeneous broadening of the medium, allows us to minimize the $\tilde{\varepsilon}'(\omega) << \tilde{\varepsilon}''(\omega)$ summand in (2) and largely restore the RT-symmetric properties of the medium for a quasi-monochromatic pulse [17, 20, 24].

The boundary value problem of dynamic Bragg diffraction is solved by the spectral method near the Bragg condition in the two-wave approximation [20, 24]. The field impulse (1) incident on the FC is represented as a Fourier integral

$$E_{in}(x,t) = \int_{-\infty}^{\infty} A_{in}(\Omega) \exp(ik_x x - i\omega t) d\omega \,, \tag{3}$$

where $A_{in}(\Omega) = \dfrac{1}{2\pi} \int_{-\infty}^{\infty} A_{in}(t') \exp(i\Omega t') dt'$ is the spectral amplitude of the incident pulse,

$, t' = t - x \sin\theta / \mathrm{c} \; \Omega = \omega - \omega_0$ . From the Helmholtz equation

$$\Delta E(\vec{r},\omega) + \varepsilon(z,\omega) k^2 E(\vec{r},\omega) = 0 \,, \tag{4}$$

where $k = \omega / c$, and boundary conditions at the boundaries of the FC $z = 0, L$ for each spectral field component $E(x,z,\omega)$ in the FC in the two-wave approximation (near the Bragg condition) are analytically found the spectral amplitudes of direct $A_0(\Omega)$ and diffracted $A_h(\Omega)$ waves and the corresponding dispersion relations for the *z-projections* of the wave vectors $q_{0z}(\Omega)$ and $q_{hz}(\Omega) = q_{0z}(\Omega) - sh$ inside the FC:

$$q_{0z}^{(1,2)} = sh/2 \pm (k^2/h)\sqrt{\alpha^2 - \varepsilon_1\varepsilon_{-1}}\,, \qquad (5)$$

where the value $\alpha = (h/k^2)[(\varepsilon_0 k^2 - q_{0x}^2)^{1/2} - h/2]$ defines the deviation from the exact Bragg condition $\alpha = 0$ ;

$$\begin{aligned} \varepsilon_1 &= [\varepsilon' + i\tilde{\varepsilon}(\omega)]/2 = [\varepsilon' - \tilde{\varepsilon}''(\omega) + i\tilde{\varepsilon}'(\omega)]/2, \\ \varepsilon_{-1} &= [\varepsilon' - i\tilde{\varepsilon}(\omega)]/2 = [\varepsilon' + \tilde{\varepsilon}''(\omega) - i\tilde{\varepsilon}'(\omega)]/2 \end{aligned} \qquad (6)$$

- coefficients in the Fourier series expansion of function (2).

As can be seen from expressions (5) and (6), PT-symmetric field modes corresponding to real values of $q_{0z}^{(1,2)}$ , can propagate in the FC near the Bragg condition $\alpha << 1$ only if the value of $-\varepsilon_1\varepsilon_{-1} > 0$ , i.e., at a sufficiently large gain in the medium when $\tilde{\varepsilon}''(\omega) > \varepsilon'$ . The inequality $\tilde{\varepsilon}''(\omega) = \varepsilon'$ corresponds to a special point of spontaneous decay of PT-symmetric solutions at a certain frequency $\omega_0$ , when the real part of the resonant permittivity $\tilde{\varepsilon}'(\omega) = \tilde{\varepsilon}'(\omega_0) = 0$ .

The fields of the direct $E_0(x,z,t)$ and diffracted $E_h(x,z,t)$ waves at any point in the medium at each time instant are calculated using Fourier synthesis:

$$E_g(x,z,t) = \\ = \int_{-\infty}^{\infty} \{A_{g1}(\Omega)\exp[i(q_{0z}^{(1)} - sf)z] + A_{g2}(\Omega)\exp[i(q_{0z}^{(2)} - sf)z]\}\exp(iq_{0x}x - i\omega t)d\Omega\,, \qquad (7)$$

where $g = 0, h$ are indices referring to the transmitted and diffracted waves, respectively; $f = 0$ , if $g = 0$ , and $f = h$ , if . $g = h$

The wave amplitudes in the FC $A_{01,02}$ are found from the boundary conditions. When the pulse falls on the left surface $z=0$, i.e., at $:k_{0z}>0$ $E_0(z=0)=A_{01}+A_{02}=A_{in}(\Omega)$ and $E_h(z=L)=R_1A_{01}\exp(iq_{0z}^{(1)}L)+R_2A_{02}\exp(iq_{0z}^{(2)}L)=0$. Hence, the following expressions for the amplitudes $A_{01,02}$ :

$$A_{01}=\frac{A_{in}(\Omega)}{1-P},\quad A_{02}=-\frac{PA_{in}(\Omega)}{1-P} \tag{8}$$

where $P=(R_1/R_2)\exp(i2\varphi),\ 2\varphi=(q_{0z}^{(1)}-q_{0z}^{(2)})L$, $R_{1,2}=-\left(\alpha\mp s\sqrt{\alpha^2-\varepsilon_1\varepsilon_{-1}}\right)/\varepsilon_{-s}$

In the case of impulse falling on the right surface of FC $z=L$, i.e. at $k_{0z}<0$ and $q_{0z}<0$, the boundary conditions have the form, $E_0(z=L)=A_{01}\exp(iq_{0z}^{(1)}L)+A_{02}\exp(iq_{0z}^{(2)}L)=A_{in}(\Omega)$ $E_h(z=0)=R_1A_{01}+R_2A_{02}=0$, whence

$$A_{01}=\frac{A_{in}(\Omega)}{1-P}\exp(-iq_{0z}^{(1)}L),\ A_{02}=-\frac{PA_{in}(\Omega)}{1-P}\exp(-iq_{0z}^{(2)}L) \tag{9}$$

Since in the case of wave packet collapse $k=k_0+\Omega/c$ and $k_{0x}=k\sin\theta$, the detuning parameter $\alpha$ has the following explicit form $\alpha=\alpha(\Omega)=(k_0+\Omega/c)\sqrt{\varepsilon_0-\sin^2\theta}-h/2$.

The spectral amplitude coefficients of the transmission $T(\Omega)=A_0(L,\Omega)/A_{in}(\Omega)=[A_{01}(\Omega)e^{iq_{0z}^{(1)}L}+A_{02}(\Omega)e^{iq_{0z}^{(2)}L}]/A_{in}(\Omega)$ and reflection $R_l(\Omega)=A_h(0,\Omega)/A_{in}(\Omega)=[A_{h1}(\Omega)+A_{h2}(\Omega)]/A_{in}(\Omega)$ fields when radiation falls on the left ($k_{0z}>0$) surface $z=0$ of the FC are of the form (here $A_{hj}=R_jA_{(0)(j)}, s=1$ ):

$$T(\Omega)=\frac{1}{1-P}\left(1-\frac{R_1}{R_2}\right)\exp(isq_{0z}^{(1)}L),$$
$$R_{l,r}(\Omega)=\frac{R_1}{1-P}\left[1-\exp\{is(q_{0z}^{(1)}-q_{0z}^{(2)})L\}\right]. \qquad (10)$$

The case of radiation falling to the right ($k_{0z}< 0$) corresponds to the value $s=-1$ in (10).

## RESULTS AND DISCUSSION

For convenience of further analysis let's write down spectral amplitudes coefficients of reflecting $R(\Omega)$ and passing $T(\Omega)$ FC in a different form:

$$T(\Omega)=\frac{W}{W\cos\varphi-i\alpha\sin\varphi}, \qquad (11)$$

$$R_{l,r}(\Omega)=i\frac{\varepsilon'(1\mp\sigma)\sin\varphi}{2[W\cos\varphi-i\alpha\sin\varphi]}=i\frac{\varepsilon'(1\mp\sigma)\sin\varphi}{2W}T(\Omega), \qquad (12)$$

where

$$W=\sqrt{\alpha^2-\varepsilon_1\varepsilon_{-1}}, \qquad (13)$$

$$\varphi=k^2WL/h, \qquad (14)$$

indices "$l,r$ " and signs "-" and "+" in (12) correspond to reflection coefficients at radiation incident on the left $z=0$ and right $z=L$ boundaries of the FC. The value $\alpha=\alpha_\Omega-\alpha_\theta$ determines the deviation from the exact Bragg condition in terms of the angle $\Delta\theta=\theta-\theta_B=0$ and frequency $\Omega=0$, where $\alpha_\Omega=2(\Omega/\omega_0)\varepsilon_0\cos^2\theta_B$, $\alpha_\theta=\Delta\theta\varepsilon_V\sin 2\theta_{BV}$, $\cos\theta_B=\lambda_0/2d\sqrt{\varepsilon_0}$,

$\sin\theta_{BV} = \sqrt{\varepsilon_0/\varepsilon_V}\sin\theta_B$ $\varepsilon_V$ are the dielectric constant of the medium surrounding the FC.

Note that the transmission coefficient $T(\Omega)$ (11) is independent of the direction of radiation incident on the FC, whereas the reflection coefficient $R(\Omega)$ (12) varies significantly up to , $R_l(\Omega=0)=0$ $R_r(\Omega=0) >> 1$ - unidirectional reflection.

In the case of broadband quasi-RT symmetry, when the material dispersion is small, i.e. $\tilde{\varepsilon}'(\omega) \approx \tilde{\varepsilon}'(\omega_0) = 0$ it follows from (6) that

$$-\varepsilon_1\varepsilon_{-1} = (\tilde{\varepsilon}''^2 - \varepsilon'^2)/4 = \varepsilon'^2(\sigma^2-1)/4, \tag{15}$$

where the value $\sigma = \tilde{\varepsilon}''/\varepsilon'$ characterizes the proximity to OT $\sigma=1$ .

From expressions (7), (11), and (12), we can see that to find the time dependence of the fields of the passed $E_{Tl}(L,t) = E_{Tr}(0,t)$ and reflected $E_{Rl}(0,t)$, $E_{Rr}(L,t)$ pulses, it is necessary to calculate the following Fourier transform integrals for the passed and reflected pulses ( $x=0$ ):

$$E_T(t) = \int_{-\infty}^{\infty} \frac{W}{W\cos\varphi - i\alpha\sin\varphi} A_{in}(\Omega)\exp(-i\omega t)d\Omega,$$
$$E_{Rl,r}(t) = \int_{-\infty}^{\infty} i\frac{\varepsilon'(1\mp\sigma)\sin\varphi}{2[W\cos\varphi - i\alpha\sin\varphi]} A_{in}(\Omega)\exp(-i\omega t)d\Omega. \tag{16}$$

Phase (14)

$$\varphi = k^2WL/h = (k^2Nd/h)\sqrt{\alpha^2 + \varepsilon'^2(\sigma^2-1)/4} \tag{17}$$

varies with the thickness $L = Nd$ of the photonic crystal and the proximity parameter to the OT $\sigma$ , here $N$ is the number of periods.

As can be seen from expressions (11), (13)-(15), exactly in the OT, i.e., at $\sigma = 1$ , the passage coefficient (11) is equal to

$$T(\Omega) = \frac{1}{\cos\varphi - i(\alpha/\sqrt{\alpha^2})\sin\varphi} = \frac{1}{\cos\varphi \pm i\sin\varphi}, \tag{18}$$

i.e. $|T(\Omega)| = 1$ at any valid $\alpha$ . When deviating from the OT, $\sigma \neq 1$ , near the Bragg condition $|\alpha| << 1$ , the functions $T(\Omega)$ and $R(\Omega)$ have spectral singularities, or poles of the functions (zeros in the denominators). Indeed, in the simplest case , $\alpha = 0$ $W \neq 0$ in (11) we obtain $T(\Omega) = 1/\cos\varphi$ . Thus, there appear frequency singularities $T(\Omega_j), R(\Omega_j) \to \infty$ at frequencies $\Omega_j$ at phase values of

$$\varphi = (\pi/2)(2m+1), \tag{19}$$

where $m = 0, 1, 2, ...$

In the presence of singularity, for correct calculation of integrals in (16) it is necessary to pass to integration in the complex plane of the complex variable, i.e., to complex frequencies $\Omega = \Omega' + i\Omega''$ . As it is known [20, 26], it follows from the principle of causality that in the integral

$$\chi(z,\omega) = \int_0^\infty \tilde{\chi}(z,\tau')\exp(i\omega\tau')d\tau', \tag{20}$$

relating the complex dielectric susceptibility $\chi(z,\omega)$ and the real Green's function $\tilde{\chi}(z,\tau')$ , the response delay time of the system $\tau'$ is a positive value, $\tau' > 0$ . Hence, in the case of analytic Green's function and complex frequency $\omega = \omega' + i\omega''$ , the function $\chi(z,\omega)$ in (20) will also be analytic if $\mathrm{Im}\,\omega > 0$ , i.e., in the upper complex half-plane of the complex variable. Thus, the principle of causality can be observed only if $\Omega'' > 0$

, but this is a necessary but not a sufficient condition for the principle of causality to hold. The poles of functions in the $\Omega'' > 0$ field when calculating integrals (16) should be taken into account in such a way that the integration contour does not include these poles. In this case, the fields $E_T(t), E_R(t)$ will not increase in the area $t < 0$ . In other words, the integration contour in (16) should be chosen above the poles $\Omega''_j$ of the functions , $T(\Omega)$ $R(\Omega)$ , Fig. 2, or, in addition to integration along the real axis $\Omega = \Omega'$ , it is necessary to bypass the poles and calculate the sums of deductions [27-29]:

$$E_T(t) = \int_{-\infty+i\gamma}^{\infty+i\gamma} T(\Omega) A_{in}(\Omega) \exp(-i\omega t) d\Omega ,$$
$$E_{Rl,r}(t) = \int_{-\infty+i\gamma}^{\infty+i\gamma} R_{l,r}(\Omega) A_{in}(\Omega) \exp(-i\omega t) d\Omega, \qquad (21)$$

where , ,$\Omega = \Omega' + i\gamma$ $\gamma > \Omega''_j$ $-\infty < \Omega' < +\infty$ . Failure to fulfill this requirement leads to violation of the principle of causality - the reflected radiation appears before the incident pulse arrives at the medium. If the poles are in the lower half-plane, $\Omega''_j < 0$ , then integration along the real axis is sufficient $\Omega = \Omega'$ .

It follows from (17) and (19) that the first singularity point at $\varphi = \pi/2$ corresponds to the critical value of FC thickness at the center frequency $L_{cr} = \pi h / k_0^2 \sqrt{4\alpha^2 + \varepsilon'^2(\sigma^2 - 1)}$ . Analytical and numerical calculations have shown that at $L > L_{cr}$ the poles $\Omega''_j$ of the functions $T(\Omega)$ and $R(\Omega)$ lie in the upper half-plane, i.e., in the lower half-plane. $\Omega''_j > 0$, and at $L < L_{cr}$ - in the lower half-plane, i.e., $\Omega''_j < 0$ . The case $L > L_{cr}$ corresponds to the process of laser generation when the field intensity in the FC increases rapidly in time. This leads to nonlinear interaction of radiation with matter, which is not described in our linear model.

Fig. 3 shows plots of the moduli of the reflection spectra of pulses $R_r(\Omega')$ (12) at different parameters of proximity to the OT$\sigma$, with all values of $\sigma > 1$, i.e., RT-symmetric modes propagate in the FC. From the comparison of the plots, it can be seen that at $\sigma = 1.1$ the spectrum of the reflected pulse is significantly narrower not only compared to the spectrum of the incident pulse, curve *4*, but also with respect to the spectra at other close values of $\sigma = 1.11;\ 1.09$. The magnitude of the reflection coefficient at $\sigma = 1.1$ increases manifold. This is due to the appearance of a frequency singularity in the FC, since at the given value of $\sigma$ and the chosen number of periods $N = 109$ the phase value in (11), (12) is close to the critical value $\varphi \approx \pi/2$.

The narrowing of the spectra of reflected and passed pulses under conditions of frequency singularity must inevitably lead to the corresponding delay of pulses in time. The parameters of the problem are chosen so that the condition $L < L_{cr}(\sigma)$, i.e., the poles of the functions $R_{r,l}(\Omega)$ and $T(\Omega)$ lie in the lower half-plane for the FC. Therefore, to determine the time dependence of the reflected and passed pulse intensities, we can perform integration in (16) only along the real frequency axis. Fig. 4 shows plots of the intensities of the reflected $I_{Rl,(r)}(t) = |E_{(Rl)(,)r}(t)|^2$ and passed $I_T(t) = |E_T(t)|^2$ pulses at different values $\sigma$. From comparing the plots, it can be seen that a small change of $\sigma$ by 1% leads to a multiple increase in pulse duration and gain. Compared to the incident pulse, the duration of the reflected signal increases more than 20 times. Similar significant changes in pulse intensity and duration are also observed when the crystal thickness varies $L$ near the critical value $L_{cr}(\sigma)$ with a constant value $\sigma$ . Further increase of $L > L_{cr}(\sigma)$ leads to the appearance of function poles in the upper half-plane of complex frequencies, and the transition to the mode of laser generation of radiation in FC occurs.

It is important to note that, similar to the case of monochromatic radiation, a typical PT-symmetric effect - unidirectional Bragg reflection - is observed for the short pulses under consideration. Thus, from the comparison of the plots in Fig. *4a* and Fig. *4b*, we can see that when the sign of the angle of incidence of the pulse on the structure changes, the intensity of the reflected pulse radically decreases: $I_{(Rl)}(t) = I_{(Rr)}(t)/440$. At the same time, the intensity of the passing pulse $I_{(T)}(t)$ does not change.

Let us compare the obtained results with the case of interaction of radiation with a conservative FC (without amplification and absorption) of finite thickness. It is known that a transmission resonance [30] (or thickness oscillations) is observed at the edge of the photonic forbidden zone in such a PC, when radiation in a narrow frequency range is not reflected from the PC, $R = 0$ , and the transmission coefficient is $T = 1$ . This effect is related to the interference of backward (forward) Bloch waves in the crystal and is observed when the phase difference of two Bloch waves $(q_{0z}^{(1)} - q_{0z}^{(2)})L = \pi$ . In PT-symmetric FC at such a phase difference of PT- symmetric modes, a spectral singularity and a significant growth of $R, T \to \infty$ .

CONCLUSION

The above-described supermonochromatization and amplification of short pulses in RT-symmetric FCs due to frequency singularity at certain values of the parameter of proximity to the special point $\sigma$ are also preserved in the case of dispersion at broadband RT-symmetry of the medium. For short pulses in a dispersing medium, there is also an asymmetry of pulse reflection at the change of sign of the Bragg angle of incidence of radiation on the FC. Since the value of $\sigma$ is determined by the real and imaginary parts of the dielectric permittivity, the sharp dependence of the reflection and transmission spectra of short pulses on $\sigma$ detected near the frequency singularity can be used in the development of new physical principles of creating devices for controlling the parameters of short optical pulses, power limiters, optical sensors, etc.

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FIGURE CAPTIONS

**Fig. 1.** Even (curve *1*) and odd (curve *2*) distribution functions of the real and imaginary parts of the resonant permittivity in a PT-symmetric photonic crystal, dashed line *3* - $\varepsilon_0$; pulse incidence patterns (b) on the left ($k_{(0)(z)>0}$) and right ($k_{(0)(z)<0}$) on the FC.

**Fig. 2**. Illustration of two possible paths when integrating functions with spectral singularity: integration along the real frequency axis at $\Omega'' > \Omega_j$ and along a path with a pole bypass at $\Omega_j$.

**Fig. 3**. Reflection spectra $R_r(\Omega')$ at different values of the parameter $\sigma$ : *1* - 1.09, *2* - 1.10, *3* - 1.11; *4* - spectrum of the incident pulse $A_{in}(\Omega')$, the pulse falls to the right ($k_{0z} < 0$). Parameters: $N = 109$, $\lambda_0 = 0.8$ μm, $d = 0.5$ μm, $\varepsilon_0 = 1.3$, $\varepsilon' = 0.0254$, Gaussian pulse duration $\tau = 0.1$ ps.

**Fig. 4**. Intensities of reflected $I_{(Rr)(,)(l)}(t)$ (red curves *1* and *2*), passed $I_{(T)}(t)$ (blue curves *3* and *4*) and incident $I_{(in)}(t)$ pulses (curves *5*, right scale) for FC with $N = 109$ at values of the proximity parameter to the OT $\sigma = 1.10$ (curves *1* and *3*) and $\sigma = 1.09$ (curves *2* and *4*): the pulse drops to the right (a) ($k_{0z} < 0$) and to the left (b) ($k_{0z} > 0$). The rest of the parameters are as in the caption of Fig. 3.

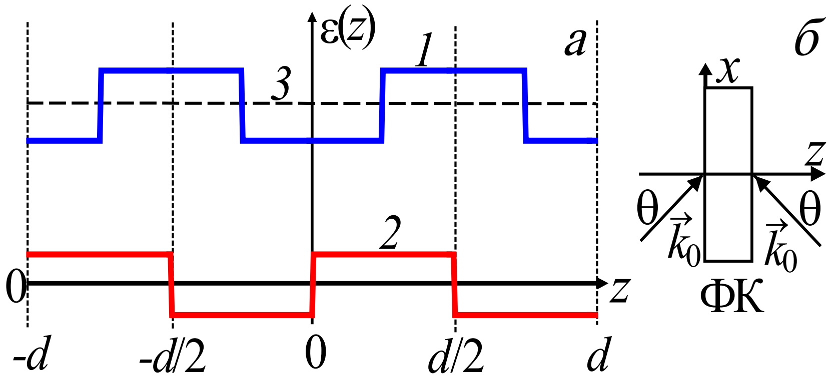


Fig. 1.

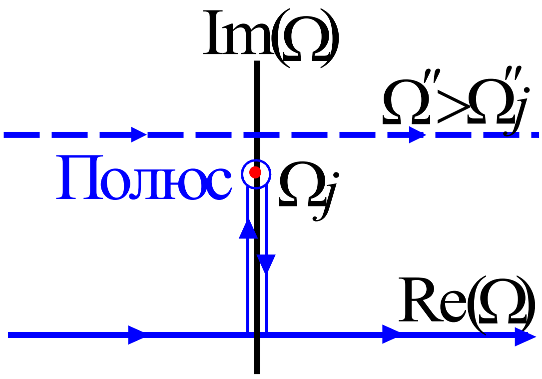


Fig. 2

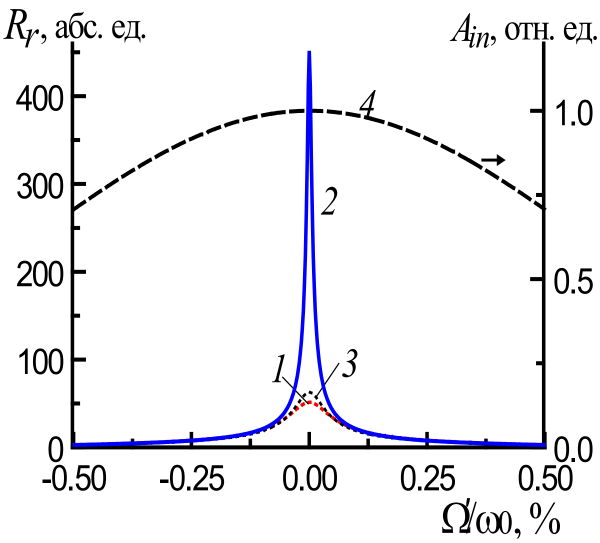


Fig. 3.

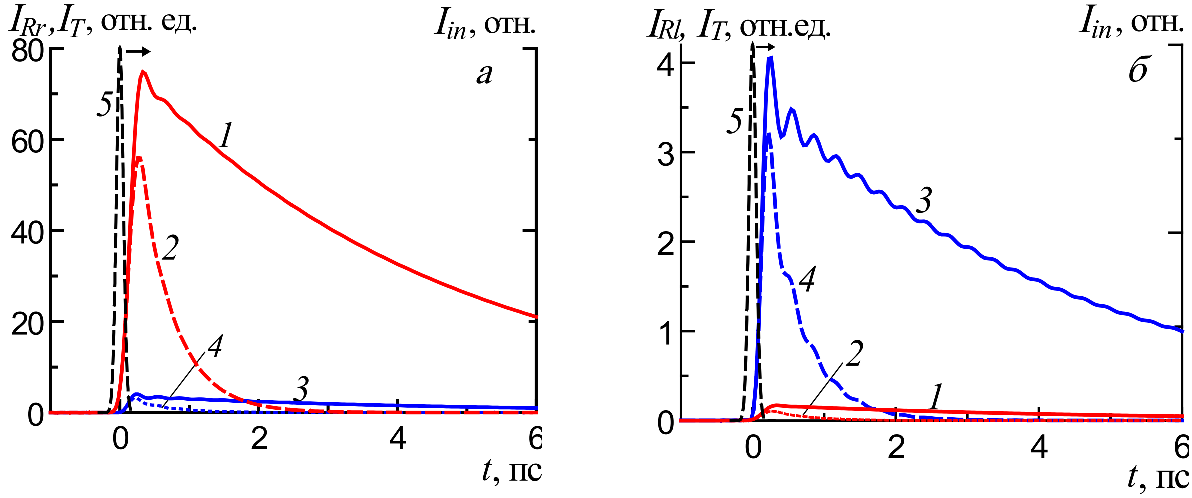
$I_{Rr}, I_T$, отн. ед.
$I_{in}$, отн.
a
80
60
40
20
0
0
2
4
6
$t$, пс
1
2
3
4
5
$I_{Rl}, I_T$, отн.ед.
$I_{in}$, отн.
б
4
3
2
1
0
0
2
4
6
$t$, пс
1
2
3
4
5

Fig. 4.