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# PHOTON SPLITTING PROCESS IN A STRONG MAGNETIC FIELD CONSIDERING THE INFLUENCE OF POSITRONIUM


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**Abstract.** The process of photon splitting $\gamma \to \gamma\gamma$ in a strong magnetic field considering the contribution of positronium to photon dispersion has been examined. It is shown that such conditions lead to the opening of a new reaction channel and changes in the selection rules for photon polarizations. The corresponding partial probabilities for allowed channels have been calculated. An estimate of the efficiency of the process under consideration has been obtained.

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## 1. INTRODUCTION

The photon splitting process $\gamma \to \gamma\gamma$, forbidden in vacuum by the charge parity conservation law known as Furry's theorem, becomes allowed in the presence of an external field and/or plasma, which change both the interaction amplitude and the dispersive properties of photons. Despite the long history of research, interest in this process continues due to its possible astrophysical applications.

Apparently, the probability of photon splitting in a weak magnetic field[1)] ($B \ll B_e = m^2/e \simeq 4.41 \cdot 10^{13}$ G) and at low photon energies ($\omega \ll m$) was first calculated in works [1,2]. In a more detailed study [3], polarization selection rules were obtained, and it was shown that in the limit of collinear kinematics, there will be only one splitting channel out of two possible ones. In the same work, the photon splitting amplitude was calculated at initial photon energy below the electron-positron pair ($\omega \leqslant m$) production threshold for magnetic field of arbitrary strength.

One of the manifestations of this process in astrophysics is the explanation of the softening of radiation spectra from magnetized neutron stars. In particular, it is assumed that it can be used to establish the nature of gamma-ray spectral features of some radio pulsars [4]. In works [5, 6], models were proposed explaining the cutoff in the spectra of soft repeating gamma-ray burst sources due to the influence of the process $\gamma \to \gamma\gamma$.

Another interesting application of the considered process is explaining the radio emission characteristics of anomalous X-ray pulsars and soft gamma repeaters. Since the photon splitting reaction has no kinematic threshold, energetic photons propagating at small angles relative to the magnetic field can split before reaching the pair production threshold. Thus, the process can alter the efficiency of electron-positron plasma production necessary for creating the observed radio emission [7–9].

Photon splitting should also be considered in models of flare activity in soft gamma repeaters [5, 10]. The process in question can act as a regulator of temperature in the outer photosphere of a long-lived region occupied by magnetically trapped hot $e^+e^-$-plasma and photons in thermodynamic equilibrium with it (so-called trapped fireball).

In all considered astrophysical applications, the reaction $\gamma \to \gamma\gamma$ occurs in the presence of a strong external magnetic field and plasma, which may have high density and temperature. Investigation of the process $\gamma \to \gamma\gamma$ in a strong magnetic field, taking into account that photon dispersion leads to significant deviations from collinear kinematics, was conducted

[1] The paper uses natural system of units: $c = \hbar = k = 1$, electron mass, $e > 0$ – elementary charge.

in work [11]. The plasma influence on photon splitting probability manifests in two ways. On the one hand, it modifies the dispersion properties of photons, on the other hand it changes the process amplitude. The first factor was considered in works [3, 12]. It was shown that the presence of cold and weakly magnetized plasma does not change the selection rules if the plasma density is not too high $n_e \leqslant 10^{19}$ cm$^{-3}$ [3]. In work [12], the process probability was calculated taking into account the plasma influence on photon dispersion, but using amplitudes obtained in the presence of a weak magnetic field without plasma. It was shown that with this approach, the plasma influence is negligible everywhere except for a very narrow range of plasma and magnetic field parameters.

The change in the photon splitting amplitude in the presence of a magnetic field and plasma was considered in works [13, 14] based on the effective Euler-Heisenberg Lagrangian, taking into account thermal corrections in one- and two-loop approximations. It was shown that in the low-temperature limit, the process $\gamma \to \gamma\gamma$ can compete with other absorption reactions, such as the Compton effect.

Another approach was considered in work [15], where the amplitudes and absorption coefficient of photon splitting in the strong field limit were calculated taking into account changes in the electron propagator in a magnetic field. The main conclusion was that the plasma influence is negligibly small. Nevertheless, the absorption coefficient estimates obtained there are inaccurate in the high-energy limit, as the expressions used are only applicable for low-energy approximations. However, works [13–15] did not consider effects related to plasma influence on photon dispersion properties.

Joint consideration of the magnetized medium's influence on both the photon splitting amplitude and changes in its dispersion properties was conducted in works [16, 17]. They obtained modified polarization selection rules: the channel forbidden in the absence of plasma became open, and the probabilities in channels allowed in pure magnetic field (magnetized vacuum) were suppressed. Furthermore, they compared the probabilities of photon splitting and fusion processes with Compton scattering and showed that the photon splitting reaction can compete with Compton scattering under conditions of rarefied plasma ($T \ll m$) and as a result can contribute to the formation of spectra of anomalous X-ray pulsars and soft gamma repeaters.

In addition to pure magnetic field and plasma, the influence of bound states, such as positronium, can be an additional factor modifying the photon dispersion properties. At first glance, its contribution to the photon polarization operator should be of the next order of smallness in the fine structure constant $\alpha$, however, in a strong magnetic field, it leads to a significant change in the photon dispersion properties near the cyclotron resonance [18], which, in turn, affects the probability of the process itself. In particular, paper [19] investigated the influence of positronium on the dispersion properties and amplitudes of the neutrino radiative decay process, $\nu \to \nu\gamma$, in a strong magnetic field. It was shown that accounting for effects associated with positronium leads to a significant increase in the probability of this process. It is natural to expect that such influence of positronium on photon dispersion will lead to changes in the probability of photon splitting process.

The paper is structured as follows. In Section 2, using previously obtained results for photon dispersion in a strong magnetic field considering positronium influence, new selection rules for the photon splitting reaction are derived. Section 3 presents the results of numerical calculations of the process probability $\gamma \to \gamma\gamma$ in a strong magnetic field, taking into account positronium influence on photon dispersion for allowed channels. Section 4 evaluates the efficiency of polarized photon production through the considered process. The Conclusions provide a discussion of the obtained results.

## 2. PHOTON DISPERSION IN A STRONG MAGNETIC FIELD CONSIDERING POSITRONIUM INFLUENCE

It is well known that radiation propagation in any active medium is conveniently described in terms of normal modes [17], i.e., in the language of eigenfunctions $\varepsilon_\alpha$ and eigenvalues $\varkappa$ of the polarization operator, which determine the polarization and dispersion properties of the photon respectively.

In a pure magnetic field, physically observable photons will have only two polarization states [20], corresponding to polarization vectors[2)]

$$\varepsilon_\alpha^{(1)}(q) = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)}(q) = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}} \tag{1}$$

with a specific dispersion law (see, for example, papers [11, 20]).

On the other hand, accounting for the contribution of bound states can change this situation. In particular, as shown in work [18], in a strong magnetic field ($B \gg B_e$), the contribution of excited positronium states to the photon polarization operator is suppressed relative to the contribution of the ground state by a factor

[2] The designation of polarization vectors by symbols 1 and 2 corresponds to $\parallel$- and $\perp$-polarizations in [3] or *E*O- and *O*X-modes in charge-symmetric plasma [10].

of order $B / B_e\alpha^2 \gg 1$. In this case, the production of positronium in the ground state by a mode 1 photon or single-photon decay to it is kinematically forbidden, thus, the dispersion law for the mode 1 photon remains practically unchanged and stays close to vacuum, while the 4-momentum vector is space-like, $q^2 \lesssim 0$ [11].

However, for mode 2 photon, this conclusion changes dramatically. The dispersion law for mode 2 photon is determined from equation $q^2 - \varkappa^{(2)} = 0$, where $\varkappa^{(2)}$ is the eigenvalue of the polarization operator in a strong magnetic field, which, taking into account the influence of positronium, has the following form [19]:

$$\varkappa^{(2)} = -\alpha\beta\exp(-\rho)\left(\frac{2}{\pi}H(z) + \frac{2\lambda z}{1-\lambda^2 - z}\right), \quad (2)$$

Here $\rho = q_\perp^2/2\beta$, $z = q_\parallel^2/4m^2$, function $H(z)$ is defined as follows:

$$H(z) = \frac{1}{\sqrt{z(1-z)}} \times \times \operatorname{arctg}\sqrt{\frac{z}{1-z}} - 1, \quad 0 \leqslant z \leqslant 1, \quad (3)$$

$$H(z) = -\frac{1}{2\sqrt{z(1-z)}}\ln\frac{\sqrt{z}+\sqrt{z-1}}{\sqrt{z}-\sqrt{z-1}} - -1 + \frac{i\pi}{2\sqrt{z(z-1)}}, \quad z > 1, \quad (4)$$

$$\lambda = \frac{\alpha}{2}(\ln(4.5u) - 2.44\ln(\ln(0.15u))), \quad u = \frac{\beta}{\alpha^2}\frac{\exp(E_i(-\rho))}{\rho} \gg 1, \quad (5)$$

$E_i$ is the exponential integral function:

$$E_i(-\xi) = \int_{-\infty}^{-\xi}\frac{\exp(t)}{t}dt. \quad (6)$$

The photon polarization operator, taking into account the influence of the bound electron- positron pair, allows finding the dispersion law $q_\parallel^2 = f(q_\perp^2)$ for the mode 2 photon in the vicinity of cyclotron resonance, $q_\parallel^2 = 4m^2$, where the influence of positronium is significant. The photon spectral line splits into two (Fig. 1). The upper one with increasing $q_\perp^2$ asymptotically approaches the spectral line $q_\parallel^2 = 4m^2$, corresponding to $e^+e^-$-pair with free electron and positron at rest relative to each other. The lower curve approaches the positronium spectral line $q_\parallel^2/4m^2 = 1 - \lambda^2(\rho)$ (dash-dotted line in Fig. 1).

As can be seen from formulas (2) and (3), (4), the eigenvalue of the polarization operator for a photon of mode 2 becomes large in the vicinity of cyclotron resonance. Therefore, when calculating observable characteristics (splitting probability, number of produced photons, etc.), it is necessary to take into account the renormalization of the photon wave function:

$$\varepsilon_\alpha^{(2)}(q) \to \varepsilon_\alpha^{(2)}(q)\sqrt{Z_2}, \quad Z_2^{-1} = 1 - \frac{\partial\varkappa^{(2)}(q)}{\partial\omega^2}. \quad (7)$$

The modified polarization operator, taking into account the positronium contribution, determines new selection rules for photon polarizations for the process $\gamma \to \gamma\gamma$. As analysis shows, for the upper dispersion branch of mode 2 photon (see Fig. 1) at $q_\perp^2 = 0$ we have $q_\parallel^2 \simeq 1 - \lambda(0)^2 > 0$, where $\lambda$ is determined by equation (5). Consequently, there exists a kinematic region for it where $q^2 > 0$. Its presence opens a new reaction channel $\gamma_2 \to \gamma_1\gamma_1$, which was considered closed in studies that did not take into account the influence of positronium on dispersion. At the same

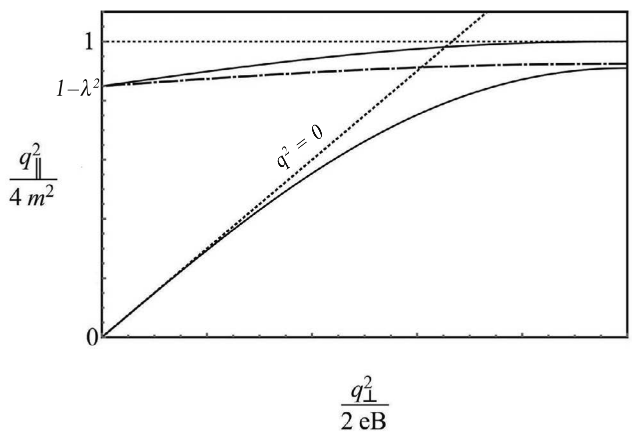


**Fig. 1.** Dispersion law for a photon of mode 2 taking into account the contribution of the bound $e^+e^-$-pair in a strong magnetic field. Solid lines - dispersion branches of mode 2 photon, dash-dotted line - positronium spectral line

time, channels $\gamma_1 \to \gamma_2\gamma_2$ and $\gamma_1 \to \gamma_1\gamma_2$ in the same region turn out to be kinematically closed.

It should be noted that a similar kinematic situation occurs for the process $\gamma \to \gamma\gamma$ in strongly magnetized plasma [17], where also in the region $q^2 > 0$ a new splitting channel opens $\gamma_2 \to \gamma_1\gamma_1$.

## 3. PROBABILITY FOR THE PHOTON SPLITTING PROCESS IN A STRONG MAGNETIC FIELD TAKING INTO ACCOUNT THE INFLUENCE OF POSITRONIUM

For various astrophysical applications and, in particular, radiation transfer analysis, it is of independent interest to calculate the probability of photon absorption in the reaction $\gamma \to \gamma\gamma$. According to work [16], the expression for it can be presented in the following form:

$$W_{\lambda\to\lambda'\lambda''} = \frac{g_{\lambda'\lambda''}}{32\pi^2\omega_\lambda}\int \left|M_{\lambda\to\lambda'\lambda''}\right|^2 Z_\lambda Z_{\lambda'} Z_{\lambda''} \times$$
$$\times\delta(\omega_\lambda(k) - \omega_{\lambda'}(k - k'') - \omega_{\lambda''}(k''))\frac{d^3k''}{\omega_{\lambda'}\omega_{\lambda''}}, \quad (8)$$

where $\lambda$ and $\lambda', \lambda'' = 1, 2$ denote the polarization of the initial photon and final photons respectively, $q_\alpha = (\omega_\lambda, k)$ – four-momentum vector of the initial photon, $q'_\alpha = (\omega_{\lambda'}, k')$ and $q''_\alpha = (\omega_{\lambda''}, k'')$ – of final photons, factor $g_{\lambda'\lambda''} = 1 - (1/2)\,\delta_{\lambda'\lambda''}$ is introduced to account for possible identity of photons in the final state.

Using the results of work [11], the photon splitting amplitudes in a strong magnetic field through allowed channels can be presented in the following form:

$$M_{1\to12} = -4\pi i\left(\frac{\varkappa}{\pi}\right)^{3/2}\frac{(q'\ q'')(q'\,\tilde{}\,q'')}{\sqrt{q_{\|}^2 q_{\perp}'^2 q_{\perp}''^2}}\times$$
$$\times H\left(\frac{q_{\|}''^2}{4m^2}\right), \quad (9)$$

$$M_{1\to22} = -4\pi i\left(\frac{\alpha}{\pi}\right)^{3/2}\frac{(q'q'')_{\|}}{\sqrt{q_{\|}^2 q_{\perp}'^2 q_{\perp}''^2}}\times$$
$$\times\left[(qq'')_{\perp} H\left(\frac{q_{\|}'^2}{4m^2}\right) + (qq')_{\perp} H\left(\frac{q_{\|}''^2}{4m^2}\right)\right], \quad (10)$$

$$M_{2\to11} = M_{1\to12}(q \leftrightarrow q''), \quad (11)$$

where function $H(z)$ is determined by formulas (3), (4).

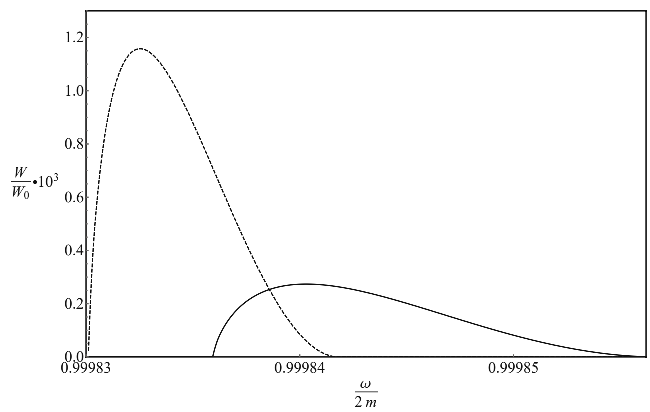


**Fig. 2.** Probability of photon absorption in channel $\gamma_2 \to \gamma_1\gamma_1$ for magnetic field values $B/B_e = 100$ (solid line) $B_3/B_e = 200$ (dashed line). Here $W_0 = (\alpha/\pi)^3 m \simeq 3.25 \cdot 10^2\,\text{cm}^{-1}$

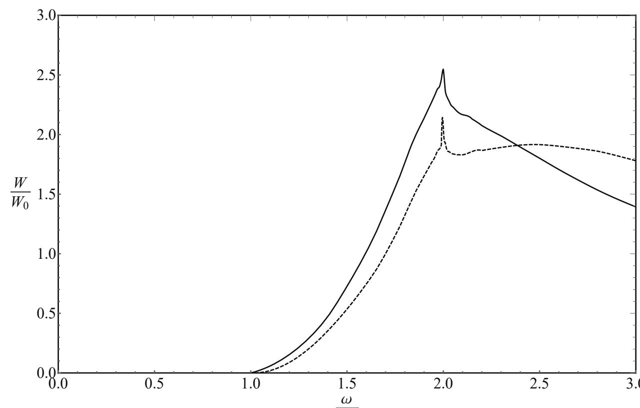


**Fig. 4.** Photon absorption probability in the channel $\gamma_1 \to \gamma_2\gamma_2$ for magnetic field values $B/B_e = 100$ (solid line $B/B_e = 200$ (dashed line). Here $W_0 = (\alpha/\pi)^3 m \simeq 3.25 \cdot 10^2\,\text{cm}^{-1}$

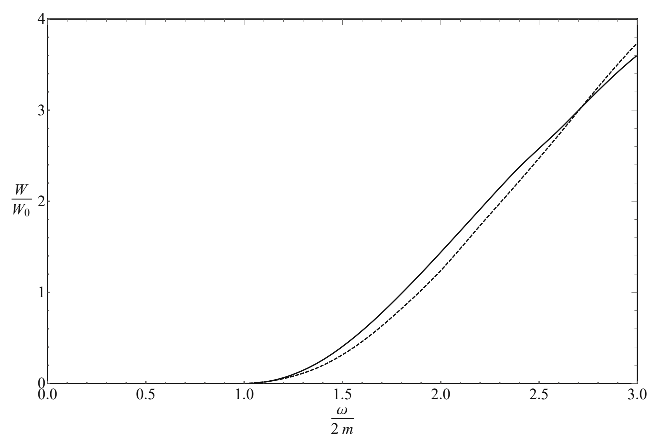


**Fig. 3.** Probability of photon absorption in channel $\gamma_1 \to \gamma_1\gamma_2$ for magnetic field values $B/B_e = 100$ (solid line), $B/B_e = 200$ (dashed line). Here $W_0 = (\alpha/\pi)^3 m \simeq 3.25 \cdot 10^2\,\text{cm}^{-1}$

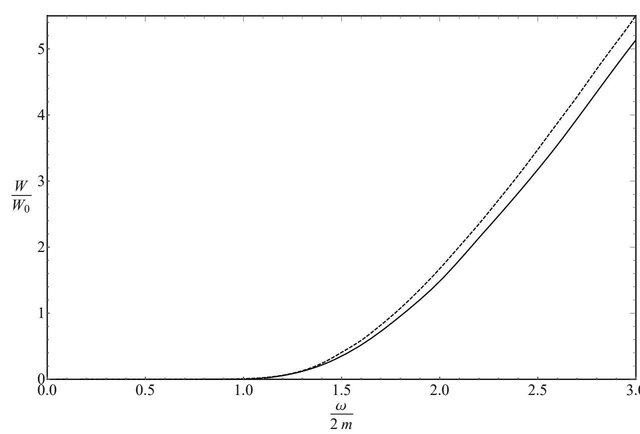


**Fig. 5.** Photon absorption probability in the channel $\gamma_1 \to \gamma_1\gamma_2$ in a strong magnetic field ($B/B_e = 200$). The dashed line corresponds to the reaction probability in magnetized vacuum without accounting for positronium contribution [11]. Here $W_0 = (\alpha/\pi)^3 m \simeq 3.25 \cdot 10^2\,\text{cm}^{-1}$

The probability of splitting in channel $\gamma_2 \to \gamma_1\gamma_1$, which is forbidden in a pure magnetic field without taking into account the positronium contribution, can be calculated analytically by analogy with the probability of photon splitting in magnetized plasma [17]. Taking into account the dispersion and renormalization of the initial photon wave function, it can be represented as

$$W_{2\to 11} = = \frac{\alpha^3}{8\pi^2} Z_2 H^2\left(\frac{q_\parallel^2}{4m^2}\right)\frac{q_\perp^2}{\omega}F\left(\sqrt{\frac{q_\parallel^2}{q_\perp^2}}\right)\theta(q^2), \quad (12)$$

where $H(z)$ is determined by formulas (3), (4), $\theta(x)$ is the theta function, and

$$F(z) = 2\ln z - 1 + z^{-2}. \quad (13)$$

The probability of photon absorption through channel $\gamma_2 \to \gamma_1\gamma_1$ is shown in Fig. 2 as a function of initial photon energy for magnetic fields $B = 100B_e$ and $B = 200B_e$.

The probabilities of photon absorption in channels $\gamma_1 \to \gamma_1\gamma_2$ and $\gamma_1 \to \gamma_2\gamma_2$ were obtained numerically from formula (8) with substitution (9) and (10) and taking into account the influence of positronium on the dispersion and renormalization of photon wave functions for magnetic field induction values $B = 100B_e$ and $B = 200B_e$. In Figs. 3–6, they are presented as functions of the initial photon energy. As seen in Figs. 5 and 6, the probability of photon splitting in these channels turned out to be lower than in a strong magnetic field without considering the positronium influence. This is due to the fact that in the kinematic region $q^2 > 0$ these channels are closed. Despite the fact that for processes involving mode 2 photons, the probability is summed over two dispersion branches, in the region where these channels are allowed, the reaction phase space still turns out to be smaller than without taking into account the positronium influence on photon dispersion.

Taking into account the influence of positronium contribution to photon dispersion in a strong magnetic field without plasma showed the existence of a non-zero decay probability through the channel $\gamma_2 \to \gamma_1\gamma_1$. In the kinematic region $\omega \leq 2m$ channel $\gamma_2 \to \gamma_1\gamma_1$ will be the main one, as channels $\gamma_1 \to \gamma_1\gamma_2$ and $\gamma_1 \to \gamma_2\gamma_2$ are kinematically suppressed. Although the probability of photon decay through channel $\gamma_2 \to \gamma_1\gamma_1$ is non-zero only in a narrow range of initial photon energies and is significantly inferior in magnitude to the contribution to probability due to Compton scattering [17], the photon decay process may play an important role in the mechanism of changing the number of photons in radiation models of soft repeating gamma-ray burst sources.

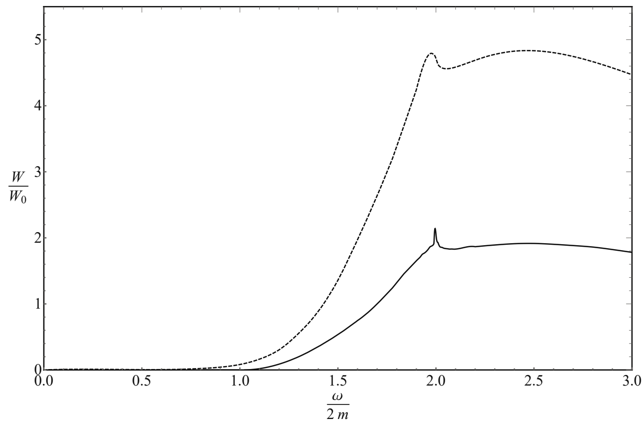


**Fig. 6.** Photon absorption probability in the channel $\gamma_1 \to \gamma_2\gamma_2$ in a strong magnetic field ($B/B_e = 200$). The dashed line corresponds to the reaction probability in magnetized vacuum without accounting for positronium contribution [11]. Here $W_0 = (\alpha/\pi)^3 m \simeq 3.25 \cdot 10^2\,\text{cm}^{-1}$

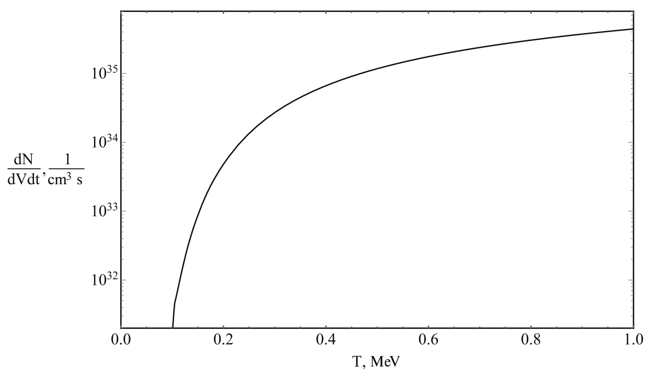


**Fig. 7.** Change in the number of mode 2 photons due to reaction $\gamma_2 \to \gamma_1\gamma_1$ in a strong magnetic field ($B/B_e = 200$) per unit volume per unit time, taken by module

## 4. EFFICIENCY OF PHOTON PRODUCTION

As noted in the Introduction, modern models describing radio emission of strongly magnetized neutron stars require efficient production of $e^+e^-$-plasma in the magnetosphere [9, 21]. In turn, electron-positron pairs in a strong magnetic field $B \gg B_e$ will most likely be produced by mode 2 [9, 22, 23] photons, therefore it is of particular interest to calculate the decrease in the number of mode 2 photons due to reaction $\gamma_2 \to \gamma_1\gamma_1$ per unit volume per unit time:

$$\frac{dN}{dVdt} \simeq -\int \frac{d^3k}{(2\pi)^3} W_{\gamma_2 \to \gamma_1 \gamma_1} f_\omega, \quad (14)$$

where

$$f_\omega = (1 + \exp(\omega/T))^{-1}$$

is the equilibrium photon distribution function. The modulus of this value as a function of temperature $T$ of the photon gas for the magnetic field value $B = 200B_e$ is shown in Fig. 7.

Numerical analysis shows that the decrease in the number of mode 2 photons in a strong magnetic field, taking into account the influence of positronium due to decay in the channel $\gamma_2 \to \gamma_1\gamma_1$, is 4–7 orders of magnitude less in the same temperature range than in the presence of strongly magnetized plasma [17]. Consequently, the influence of positronium suppresses the process of changing the number of mode 2 photons. This can be explained by the fact that in the case considered in this work, the probability of the process is strongly suppressed by the renormalization in absolute value of the photon wave function (7) due to the proximity of the kinematically allowed region of initial photon energies to the threshold $q^2\| = 4m^2$. Apparently, for a self-consistent solution of this problem, it is also necessary to take into account the presence of magnetized plasma and the influence of positronium on the reaction amplitude, which, however, is associated with significant computational difficulties.

## 5. CONCLUSIONS

In this work, we calculated the probabilities of photon absorption in the process of photon splitting into two, $\gamma \to \gamma\gamma$, in a strong magnetic field, taking into account the influence of positronium on the dispersion properties of photons. The influence of positronium on the probability of this reaction is twofold: on one hand, the dispersion line of mode 2 photon splits into two, on the other hand, the phase space of the reaction decreases. As a result, the probability of photon splitting through channels known in the presence of a magnetic field slightly decreases. Simultaneously, a new splitting channel opens $\gamma_2 \to \gamma_1\gamma_1$, which is kinematically forbidden without accounting for the positronium contribution. An expression for the probability of photon absorption in the channel $\gamma_2 \to \gamma_1\gamma_1$ was obtained in analytical form. The efficiency of changing the number of mode 2 photons due to this channel was evaluated.

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