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SELF-CONSISTENT QUASI-CLASSICAL APPROACH TO DESCRIBING PARTICLE MOTION IN A DISSIPATIVE MEDIUM

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Abstract. An approximate self-consistent approach is proposed that allows describing the quasi-classical translational dynamics of a non-relativistic particle in a dissipative medium with arbitrary dependence of the corresponding dissipative forces on velocity. It is shown that dissipation suppresses the quantum properties of the particle. This leads to the necessity of interpreting propagation in a dissipative medium as a continuous process of measuring the particle state. As examples, non-stationary coherent states of the particle are considered at three stages of its deceleration in the medium due to ionization losses. These stages correspond to high-energy losses, losses in the vicinity of the Bragg peak, and low-energy losses at the final stage of propagation.

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1. INTRODUCTION

The development of quantization methods in open systems is closely related to fundamental questions of measuring quantum object states [1, 2]. A particle can act as a quantum object, being measured by a dissipative medium acting as a classical device. In this case, continuous measurement of the particle state by the medium is accompanied by continuous collapse of the wave function [3].

Today, there are many methods for quantum-mechanical description of particle motion in dissipative media [4–17]. It should be noted that different methods often lead to somewhat different physical results and conclusions. From a physical point of view, the approach based on the Markov approximation when solving density matrix equations [18–23] appears to be quite consistent and leads to reasonable results. On the other hand, the Hamiltonian approach proposed in works [4, 5] is attractive in its simplicity. However, difficulties arise here in the quantum consideration of particle motion in bulk media [24, 25].

The Hamiltonian quantization method can be used after classical analysis of the motion of a selected particle in a medium that acts on this particle through dissipative forces. Thus, in this case, we can speak about finding a quantum correspondence to the motion of a classical particle. Already at this stage of reasoning, the quasi-classical nature of such consideration becomes clear.

In works [4–7], a quantization method is developed for the case when a non-relativistic particle is affected by a dissipative force proportional to velocity $\mathbf{v}$. In works [26, 27] a procedure for quasi-classical quantization is proposed in the presence of dissipative forces proportional to velocity and square of particle velocity. It is important to note that these forces are introduced into the theory phenomenologically, without detailed analysis of their physical nature. A consistent microscopic examination of particle motion in various media shows that the energy losses accompanying this motion can be associated with forces that depend on velocity in a very complex way [28–30]. In this regard, there arises a task of developing an approach that will allow finding a quasi-classical correspondence to the motion of a non-relativistic classical particle under the action of dissipative forces arbitrarily dependent on velocity. This work is devoted to solving this problem.

2. CLASSICAL CONSIDERATION

Let the motion of a classical particle of mass m be described by the Lagrangian L, whose explicit time dependence is determined by the function $q(t)$:

$$L = \frac{m\mathbf{v}^2}{2} q(t) \quad (1)$$

We will assume below that there are no external conservative forces. From here, using the Lagrange equation

$$\frac{d}{dt}\frac{\partial L}{\partial \mathbf{v}} = 0$$

we will come to the equation of motion

$$m\dot{\mathbf{v}} = -m\frac{\dot{q}}{q}\mathbf{v}, \quad (2)$$

where the dot above the variable denotes time derivative.

Let the dissipative force acting on the particle have the form

$$\mathbf{F}_d = -mf(v(t))\mathbf{v}, \quad (3)$$

where $f(v(t))$ is some positively defined function depending on time through velocity v of the particle.

Comparing (2) with the classical equation of motion and considering (3), we have

$$\frac{\dot{q}}{q} = f(v(t)). \quad (4)$$

Integrating (2), we get

$$\mathbf{v} = \frac{\mathbf{v}_0}{q}, \quad (5)$$

where v_0 — is the initial (at $t = 0$) velocity of the particle.

Integrating (5), we will find the classical trajectory of a particle described by the dependence of the radius vector $\mathbf{r}_c$ of the particle on time:

$$\mathbf{r}_c = \mathbf{r}_0 + \mathbf{v}_0\tau \quad (6)$$

where $\mathbf{r}_0$ is the initial radius vector of the particle, and the reduced time τ is determined by the expression

$$\tau = \int_0^t \frac{dt'}{q(t')}. \quad (7)$$

The time count here begins from the moment the particle enters the dissipative medium.

Substituting (5) into (4), we will arrive at the differential equation for determining the function q:

$$\dot{q} = qf(v_0/q). \quad (8)$$

Let us supplement this equation with the obvious (see (5)) initial condition

$$q(0) = 1. \quad (9)$$

From this and from (4) it follows that $q(t)$ is a monotonically increasing function of time. Due to dissipation, the particle eventually stops. Then, considering (5) and (9), we come to the conclusion that the values $q(t)$ lie in the interval $1 \leqslant q(t) \leqslant \infty$. From this and from (7) it can be seen that $\tau(t)$ also increases monotonically over time. If $q(t)$ reaches an infinite value in finite time t_{max}, then the upper limit in the integral (7), $t = t_{max}$, corresponds to the maximum value of the reduced time τ_{max}. Situations are possible when $t_{max} \to \infty$ (see below). In the general case, the integral (7) at $t = t_{max}$ can be either convergent or divergent.

Thus, by solving the problem (8), (9), we will find the function $q(t)$. This will determine the Lagrangian (1), which self-consistently takes into account the action of the dissipative force (3) on the particle. In turn, using (5) we will determine the classical velocity of the particle as a function of time. Then, substituting $q(t)$ into (7), we will find the reduced time τ. After this, expression (6) will allow us to determine the position of the classical particle in space at each moment of time.

From an applied point of view, it is of considerable interest to calculate the energy loss W of the particle depending on the distance traveled s. From (5) and (6) it can be seen that the motion of the classical particle is rectilinear, and its distance traveled s is determined by the expression

$$s = |\mathbf{r}_c - \mathbf{r}_0| = v_0\tau.$$

Then, moving from vector to scalar form, we write

$$\dot{v} = v\frac{dv}{ds}.$$

As a result, equation (2) taking into account (4) will take the form

$$\frac{dv}{ds} = -f(v). \quad (10)$$

Integrating this equation considering the boundary condition $v(0) = v_0$, we will find the dependence $v(s)$. From this we will obtain the dependence of energy $W = mv^2/2$ of the particle on the distance traveled. After this, we will determine the specific losses $-dW/ds$.

Obviously, the total path length of s_{max} particle during deceleration in a dissipative medium is determined by the expression $s_{max} = v_0\tau_{max}$.

Using (1), for the canonical momentum we find

$$\mathbf{p} = \frac{\partial L}{\partial \mathbf{v}} = m\mathbf{v}q. \quad (11)$$

Then for the Hamiltonian

$$H = \mathbf{p}{\cdot}\mathbf{v} - L$$

we will have

$$H = \frac{\mathbf{p}^2}{2mq(t)}. \quad (12)$$

Expressions (2), (4)–(9) describe the classical trajectory of the particle corresponding to the extremum of the action functional

$$S = \int_0^t L dt'.$$

Therefore, the solution of equation (8) with the initial condition (9) contains the value of v_0 initial particle velocity. As a result, substituting the found function $q(t)$ at a given value v_0 into expression (12) leads to an effective Hamiltonian, which self-consistently accounts for the action of dissipative forces on the classical particle that arbitrarily depend on the particle velocity. Since the nature of the explicit dependence $q(t)$ is determined only by the classical trajectory of the particle, the subsequent quantization of motion based on expressions (3), (8), (9), and (12) contains features of the semiclassical approximation [31].

The effective Hamiltonian (12) depends on the initial velocity v_0. Therefore, the considered self-consistency condition imposes a priori restriction on the form of the particle wave function. Namely, this form must contain features of a classical particle with a given initial velocity.

3. SEMICLASSICAL DYNAMICS

To transition to quantum-mechanical description, we perform a standard replacement of the canonical momentum $\mathbf{p}$ with a Hermitian operator $\hat{\mathbf{p}}$ in the coordinate representation:

$$\hat{\mathbf{p}} = -i\hbar\nabla$$

Then the non-stationary semiclassical Schrödinger equation for the wave function $\psi(\mathbf{r},t)$ can be written as

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2mq(t)}\nabla^2\psi, \quad (13)$$

where $\mathbf{r}$ is the radius vector belonging to one of the virtual particle trajectories.

It is easy to see that equation (13) leads to the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\mathbf{j} = 0$$

where

$$\rho = |\ \ |^2,$$

$$\mathbf{j} = \frac{\hbar}{2imq}(\psi^*\nabla\psi - \psi\nabla\psi^*).$$

From this, it is evident that the quantity $\rho(\mathbf{r},t)$, as in the conservative case, has the meaning of probability density of finding a particle at a point with radius vector $\mathbf{r}$ at time t.

Consequently, the normalization condition is also valid here

$$\int\rho(\mathbf{r},\, t)\, d\mathbf{r} = 1,$$

where integration is carried out over the entire space in which the particle can be located.

For the initial wave function $\psi(\mathbf{r},0)$ equation (13) has the following solution:

$$\psi(\mathbf{r},t) = \int G(\mathbf{r}-\mathbf{r}',t)\,\psi(\mathbf{r}',0)d\mathbf{r}', \quad (14)$$

where the quasi-classical Green's function $G(\mathbf{r} - \mathbf{r}',\, t)$ is defined by the expression

$$G(\mathbf{r}-\mathbf{r}',\, t) = \left(\frac{m}{2i\pi\hbar\tau}\right)^{3/2}\exp\left(i\frac{m}{2\hbar\tau}|\,\mathbf{r}-\mathbf{r}'\,|^2\right). \quad (15)$$

The dynamic parameter $q(t)$ in the quasi-classical Schrödinger equation (13) depends on the value v_0 of the initial particle velocity (see (8)). Therefore, in accordance with the self-consistent approach, we choose the initial wave function in the form of a Gaussian wave packet centered at the initial point $\mathbf{r}_0$ of the classical particle location and containing its initial velocity $\mathbf{v}_0$ [32, 33]:

$$\Psi(\mathbf{r},0) = \frac{1}{(2\pi)^{3/4}l^{3/2}}\exp\left(-\frac{|\,\mathbf{r}-\mathbf{r}_0\,|^2}{4l^2} + i\frac{m}{\hbar}\mathbf{v}_0{\cdot}\mathbf{r}\right), \quad (16)$$

where l is the initial spatial scale of probability density localization, which has the meaning of initial uncertainty in the particle's Cartesian coordinates [32, 33].

Substituting (15) and (16) into (14), we obtain

$$\psi(\mathbf{r},t) = \frac{1}{(2\pi)^{3/4}}\left(\frac{l}{l^2 + i\hbar\tau/2m}\right)^{3/2} \times$$

$$\times \exp\left[-\frac{|\,\mathbf{r}-\mathbf{r}\ \,|^2}{4(l^2+i\hbar\tau/2m)} + i\frac{m}{\hbar}\mathbf{v}_0{\cdot}\mathbf{r} - i\frac{mv_0^2}{2\hbar}\tau\right], \quad (17)$$

where $\mathbf{r}_c$ and τ are determined by expressions (6) and (7) respectively.

In a conservative medium (in the absence of dissipation) $q = 1$ at all time moments $t = 0$. Then from (7) it follows that $\tau = t$. In this case, the wave function (17) corresponds to the coherent state of a free particle [34]. For specific cases of dissipative media, when resistance forces proportional to velocity and velocity squared are present, similar states belong to the class of quasi-classical coherent states [26, 27, 35].

From (16) for probability density, we have the expression

$$\rho(\mathbf{r},t) = \frac{1}{(2\pi)^{3/2} l_\tau^3} \exp\left(-\frac{|\mathbf{r} - \mathbf{r}_c|^2}{2l_\tau^2} \right), \qquad (18)$$

where the spatial scale l_τ when the particle travels distance s at time moments $t \geqslant 0$ is determined by the relation

$$l_\tau = \sqrt{l^2 + \left(\frac{\hbar\tau}{2ml} \right)^2} = l\sqrt{1 + \left(\frac{s}{l_D} \right)^2}, \qquad (19)$$

where $l_D = 2l^2/\lambda\!\!\!^{-}$ is the diffraction length corresponding to the initial wave function (16), $\lambda\!\!\!^{-} = \hbar/mv_0$ is the initial de Broglie wavelength of the particle.

From (19) it can be seen that the probability density wave packet, as it propagates in the dissipative medium, experiences broadening up to the maximum value l_τ^{max} of its size, determined by formula (19) taking into account the substitution $s \to s_{max} = v_0\tau_{max}$. Thus, the stopping of the localized wave packet is accompanied by the "freezing" of its spatial size. At this stage in (18), the substitution $l_\tau \to l_\tau^{max}$ should be made. In this case, the radius vector of the static wave packet center is determined by the expression $\mathbf{r}_c = \mathbf{r}_0 + \mathbf{v}_0\tau_{max}$.

Under strong dissipation, $s_{max} \ll l_D$, the particle practically shows no wave properties. In this case, as follows from (19), $l_\tau^{max} \approx l$. Therefore, here the particle behaves in translational motion as a classical object.

Under weak dissipation, $s_{max} \gg l_D$, under the radical in formula (19), considering the substitution $s \to s_{max}$ only the second term can be taken into account. Then we have

$$l_\tau^{max} = \frac{\lambda\!\!\!^{-}}{2l} s_{max}. \qquad (20)$$

In this case, when describing the translational motion of the particle, it is necessary to consider its wave properties.

Using (17) and standard rules of quantum mechanics [32, 33], it is easy to show that the uncertainties Δp_j of the Cartesian components of canonical momentum, where $j = x, y, z$, do not change with time and are determined by the relations $\Delta p_j = \hbar/2l$. From this and from (11) for the uncertainties $\Delta p_j^{(ph)}$ of the Cartesian components of physical momentum $p_j^{(ph)} = mv_{j0}$ we have $\Delta p_j^{(ph)} = \hbar/2ql$. Meanwhile, the uncertainties of Cartesian coordinates $\Delta x_j = l_\tau$. Then the uncertainty relations of the "coordinate-canonical momentum" and "coordinate-physical momentum" types respectively take the form

$$\Delta x_j \Delta p_j = \frac{\hbar}{2} \frac{l_\tau}{l}, \qquad (21)$$

$$\Delta x_j \Delta p_j^{(ph)} = \frac{\hbar}{2} \frac{l_\tau}{ql}. \qquad (22)$$

From this, as well as from (9) and (19), it is evident that at the initial moment of time, the uncertainty relations (21) and (22) are minimized:

$$\Delta x_j \Delta p_j = \Delta x_j \Delta p_j^{(ph)} = \hbar/2.$$

This, as mentioned above, corresponds to the coherent state (16) of the particle at $t = 0$.

As noted above, $\tau = t$ in a conservative medium, and the wave function (17) with such substitution possesses the properties of a coherent state of a free particle [34]. In this case, the uncertainty relations (21) and (22) are identical, as there is no difference between canonical and physical momenta. Then, as seen from (19), the right-hand sides of the uncertainty relations (21) and (22) grow infinitely with time.

Based on what was said in the previous paragraph, let's call the state with wave function (17) a coherent state of a particle in a dissipative medium. In this case, the equality between canonical and physical momenta is violated. The right-hand side of the uncertainty relation (21) increases monotonically with time, reaching a maximum value

$$\Delta x_j \Delta p_j = \hbar l_\tau^{max}/2l.$$

The right-hand side of the uncertainty relation (22) behaves differently. Since over time the function $q(t)$ increases, reaching infinity at the moment of particle stopping, the right-hand side at this moment turns to zero. This happens because at the moment of particle stopping, the uncertainties of all Cartesian components of physical momentum turn to zero. This situation corresponds to continuous collapse of the wave function in momentum space. For this reason, particle propagation in a dissipative medium should be considered as a continuous process of measuring its state. Random sequential collisions of the selected particle with a large number of medium particles, considered in the averaged Markov approximation, lead to dissipative losses and simultaneously represent a continuous measurement process. The dissipative medium here plays the role of a recorder (a kind of extended photographic plate) of the particle state at each moment of time. At $s = s_{max}$ (or $t = t_{max}$) there is registration of the stopped particle in a small spatial neighborhood of the final point of the classical trajectory. The characteristic radius l_τ^{max} of this neighborhood effectively limits the spread of space points where the particle can be registered during a specific measurement act.

Considering the averaged nature of describing particle deceleration dynamics through a velocity-dependent dissipative force, speaking about particle stopping here is only conditional. At low particle velocities, irregular (Brownian) collisions with medium particles have an increasing influence on its motion. The corresponding investigation is beyond the scope of this work.

Since at $t = 0$ the canonical and physical momenta are equal to each other (see (11) and (9)), equation (20) can be rewritten as

$$\frac{\Delta x_j^{max}}{s_{max}} = \frac{\Delta p_{j0}}{p_0}. \tag{23}$$

Here $p_0 = mv_0$ is the initial momentum of the particle.

The condition for the applicability of the quasi-classical approximation in our case has the form

$$\frac{\Delta x_j^{max}}{s_{max}} \sim \frac{l_\tau^{max}}{s_{max}} \ll 1.$$

Using (20) here, we obtain

$$\lambda\!\!\!^{-} / \Delta x_j^{max} \ll 1.$$

Thus, the initial de Broglie wavelength must be significantly smaller than the uncertainties in the particle coordinates. This statement exactly coincides with the conclusion presented in work [34]. From (20) and (23), it is evident that this conclusion is equivalent to the inequality

$$\Delta p_{j0}/p_0 = \Delta v_{j0}/v_0 \ll 1,$$

where $\Delta v_{j0} \sim \hbar/ml$ is the uncertainty of the j-th Cartesian component of the initial velocity.

From (20), it is clear that the ratio $\lambda\!\!\!^{-}/l$ plays the role of the diffraction divergence angle of the probability density wave packet. The small value of this angle quite obviously corresponds to the applicability condition of the quasi-classical approximation.

4. COHERENT STATES IN A MEDIUM WITH IONIZATION LOSSES

As specific examples, let us consider three stages of charged particle propagation accompanied by its ionization losses in some medium.

4.1. Ionization deceleration under high-energy loss conditions

Let a fast but non-relativistic charged particle enter a medium where it is decelerated due to energy losses through ionization of this medium. The velocity v of the entering particle satisfies the condition $v > \alpha c$, where $\alpha = e^2/\hbar c \approx 1/137$, e is the electron charge, c is the speed of light in vacuum [36]. In this case, the non-relativistic version of the Bethe-Bloch formula for specific ionization losses of a charged particle during its passage through matter has the form [37, 38]

$$-\frac{dW}{ds} = \frac{m\sigma}{v^2}, \tag{24}$$

where

$$\sigma = \frac{4\pi Z_\alpha^2 e^4}{m_e m} Zn \ln\frac{2m_e v^2}{I},$$

Z_α and Z are the charge numbers of the considered particle and the nuclei of the matter with which the particle interacts during deceleration, n is the concentration of matter nuclei, I is the ionization energy of matter atoms, m_e is the electron mass.

Since

$$m\dot{v} = mv\frac{dv}{ds} = \frac{d\left(mv^2/2\right)}{ds} = \frac{dW}{ds},$$

the right-hand side in (24) has the meaning of the classical force acting on a particle from the medium. Moving to the vector form, we write for this force

$$\mathbf{F}_d = -m\sigma\mathbf{v}/v^3.$$

The ratio under the logarithm in the expression for σ in order of magnitude equals $(v/\alpha c)^2$. Let the characteristic particle velocity be $v \sim 10^9$ cm/c $> \alpha c \sim 0.1\, c$ [39]. At such velocities, the particle can still be considered non-relativistic. Then $2m_e v^2/I \sim 10^2$. Under these conditions, the logarithm in the expression for σ is a slow function of velocity. Therefore, with a good approximation, we can set $\sigma = \text{const}$. In this case

$$f(q/v_0) = \sigma/v^3 = \sigma q^3/v_0^3.$$

After integrating (8) taking into account (9), we have

$$q = \left(1 - t/t_{max}\right)^{-1/3}, \quad (25)$$

where

$$t_{max} = \frac{v_0^3}{3\sigma}. \quad (26)$$

From this, as well as from (7) and (6), we find

$$\tau = \frac{3}{4} t_{max} \left[1 - \left(1 - \frac{t}{t_{max}}\right)^{4/3}\right], \quad (27)$$

$$\mathbf{r}_c = \mathbf{r}_0 + \frac{3}{4}\mathbf{v}_0\, t_{max} \left[1 - \left(1 - \frac{t}{t_{max}}\right)^{4/3}\right]. \quad (28)$$

Setting in (27) $t = t_{max}$, we get

$$\tau_{max} = \frac{3\, t_{max}}{4} = \frac{v_0^3}{4\sigma}.$$

Then the total range length

$$s_{max} = \frac{v_0^4}{4\sigma}. \quad (29)$$

From (29) and (19), we find the maximum size of the localized wave packet at the moment of its stop when $t = t_{max}$.

Substituting into (10) $f(v) = \sigma/v^3$, after integration we get

$$v^2 = v_0^2\sqrt{1 - s/s_{max}}.$$

After substituting this expression into (24) for specific ionization losses, we find

$$-\frac{dW}{ds} = \frac{W_0}{s_{max}\sqrt{1 - s/s_{max}}}, \quad (30)$$

where $W_0 = mv_0^2/2$ is the initial energy of the particle.

Thus, at the particle's stopping point, its specific losses increase indefinitely, and the size of the probability density wave packet reaches its maximum value.

The unlimited growth of specific losses is due to the fact that at $v \to 0$ formula (22), where σ = const, ceases to be valid. Moreover, in this case, many assumptions under which the Bethe-Bloch formula was obtained are no longer valid [29, 39].

Under real conditions, the rapid growth of specific ionization losses in the intermediate part of the particle's range is limited by the well-known Bragg peak, after which the specific losses rapidly decrease to zero values [29, 39]. In the vicinity of the Bragg peak, the particle velocity $v \sim \alpha c$. This vicinity corresponds to the second (intermediate) stage of ionization deceleration, which we will consider below.

4.2. Ionization losses in the vicinity of the Bragg peak. "Dry friction"

In the vicinity of the maximum specific ionization losses, the value dW/ds can be approximately considered constant. As a result, for this stage, we write

$$-dW/ds = 2ma, \quad (31)$$

where a is some positive constant.

Since the dissipative force is always directed against the velocity vector, we have

$$\mathbf{F}_d = -m\mathbf{a},$$

where

$$\mathbf{a} = a\mathbf{v}/v = a\mathbf{v}_0/v_0.$$

Thus, under condition (31), the dissipative force does not depend on velocity, which in classical mechanics corresponds to "dry friction". In this case

$$f(v) = a/v = aq/v_0.$$

Then, integrating (8) with respect to (9), we find

$$q = \left(1 - at/v_0\right)^{-1}. \quad (32)$$

Substituting this expression into (7), we obtain

$$\tau = t - \frac{a}{2v_0}t^2. \quad (33)$$

From this and from (6) we have a well-known expression for the radius vector of the classical trajectory during uniformly decelerated motion:

$$\mathbf{r}_c = \mathbf{r}_0 + \mathbf{v}_0 t - \mathbf{a}t^2/2. \quad (34)$$

Thus, in the vicinity of the Bragg peak, the probability density wave packet performs uniformly decelerated motion with acceleration **a** (see (18) and (34)).

From (5) it follows that the particle, and with it the probability density wave packet (see (17)), stops its motion at $q \to \infty$. Then from (32) we arrive at the expression for the particle motion time $t_{\max} = v_0/a$. From this and from (33) we find $\tau_{\max} = v_0/2a$. Thus, after traveling for time $t = t_{max} = v_0/a$ and until complete stop the distance $s_{max} = v_0^2/2a$, the probability density wave packet acquires the maximum static size, determined by formula (19) when replacing $s \to s_{max}$.

The considered stage of "dry friction" is the shortest of the three stages of ionization deceleration, as it corresponds to a small vicinity near the maximum value of specific losses.

Under real conditions, before stopping, the particle transitions to the third (low-energy) stage of ionization deceleration, where $v \ll \alpha c$ [36]. Below we will analyze the quasi-classical dynamics of the particle at this stage.

4.3. Ionization deceleration under conditions of low-energy losses. "Viscous friction"

In this case, for specific losses we have the expression [36, 40–42]

$$-dW/ds = 2m\gamma v, \tag{35}$$

where γ is a positive constant.

From (35), we conclude that in this case, the corresponding classical dissipative force (the "viscous friction" force) is proportional to velocity: $\mathbf{F}_d = -m\gamma\mathbf{v}$. From this and from (3) we have $f(v) = \gamma$. Then from (8) and (9) we find $q = e^{\gamma t}$. Note that this dependence $q(t)$ exactly coincides with the similar dependence found in works [4, 5] without using the quasi-classical approximation. Substituting this expression into (7), we find

$$\tau = \frac{1 - e^{-\gamma t}}{\gamma}. \tag{36}$$

From this and from (6) we obtain

$$\mathbf{r}_c = \mathbf{r}_0 + \mathbf{v}_0\frac{1 - e^{-\gamma t}}{\gamma}. \tag{37}$$

Substituting (36) into (19), we arrive at the expression for the spatial scale of the wave packet of probability density (uncertainty of particle coordinates) at any time. The maximum value $l_\tau^{¿}$ of this scale is determined by formula (19) taking into account the substitution

$$s \to s_{max} = v_0\tau_{max} = v_0/\gamma$$

and is achieved at $t \to \infty$, when the particle completes a full path in the medium. At this point, the probability density wave packet stops, having the form of a three-dimensional localized static domain.

After substituting $f(v) = \gamma$ into (10), subsequent integration and simple transformations, we will have for the specific losses

$$-dW/ds = 2m\gamma v_0(1 - s/s_{max}). \tag{38}$$

Thus, the specific losses decrease with increasing traveled distance and disappear at $s = s_{max}$.

All three considered stages of particle propagation in a medium with ionization losses correspond to classical resistance forces, the general expression for which has the form (3). At the initial, high-energy stage of particle deceleration, the resistance force is inversely proportional to the square of velocity. In the vicinity of the Bragg peak, the resistance force is practically independent of the velocity magnitude, therefore, by analogy with classical mechanics, we called it the "dry friction" force here. The final, low-energy stage of deceleration is described by a dissipative force proportional to the particle velocity, which corresponds to the "viscous friction" force in classical mechanics. It is clear that in all cases, we are not talking about friction forces in the physical sense, as this concept itself relates to the physics of macroscopic objects. It would be more correct here to speak about the corresponding mathematical approximation.

For a complete quasi-classical description of particle deceleration over the entire propagation distance, apparently, all three stages should be properly "stitched" together, which was not the goal of this work.

Let's provide some numerical estimates for the main stage of high-energy ionization losses. The range s_{max} of an alpha particle in air under normal conditions and initial energy $W_0 \sim 10$ MeV is about 10 cm [39]. As noted above, the given energy value corresponds to the initial velocity $v_0 \sim 10^9$ cm/c. Then the initial de Broglie wavelength for the alpha particle is $ƛ \sim 10^{-13}$ cm. Let the initial uncertainty in the alpha particle coordinate be of the order of atomic size, i.e., $l \sim 10^{-8}$ cm. Then the diffraction length $l_D = 2l^2/ƛ \sim 10^{-3}\,\text{cm} \ll s_{max}$. Consequently, in this case, the wave properties of the particle are clearly manifested. Substituting the parameter values given in this paragraph into formula (20), we will have for the uncertainty in the alpha particle coordinate after its stopping $\Delta x_j^{max} \sim l_\tau^{max} \sim 10\ ^4$ cm. Thus, after traveling through air under ionization braking conditions, the uncertainty in the alpha particle coordinate can increase by four orders of magnitude. For the time during which the travel occurs, we find $t_{max} \sim s_{max}/v_0 \sim 10^{-8}$ s. It is also useful to estimate the parameter value σ. From formula (29) we have $\sigma \sim v_0^4/s_{¿} \sim 10^{35}\,\text{cm}^3/\text{c}^4$. It is curious to note that approximately the same order of magnitude value follows from the formula written here immediately after (24). The parameter value σ can allow calculating coordinate uncertainties, range, and travel time for all other initial energies of the alpha particle.

In aluminum, the range of an alpha particle with initial energy of about 10 MeV is about 10^{-3} cm [39]. It is easy to see that in this case, the diffraction length $l_D \sim s_{max}$. Therefore, here dissipation more effectively suppresses the wave properties of the alpha particle than in air.

For the uncertainty Δv_{j0} of the initial velocity components in both examples considered above pax we have $\Delta v_{j0} \sim \hbar/ml \sim 10^4$ cm/s. Thus, the applicability condition for the quasi-classical approximation $v_0 \gg \Delta v_{j0}$ is fulfilled here with a good margin.

5. CONCLUSIONS

The approximate self-consistent approach proposed in this paper allows describing the translational dynamics of a quantum non-relativistic particle under the action of a dissipative force that depends arbitrarily on the velocity magnitude. In this sense, this approach can be considered as a generalization of the canonical method proposed by Caldirola and Kanai [4, 5], where a dissipative force proportional to the particle velocity is considered. On the other hand, the self-consistent approximation using the concept of classical trajectory leads to a quasi- classical rather than a purely quantum consideration.

In the proposed approach, the Hamiltonian (12) depends on the dynamic parameter $q(t)$. The nature of explicit time dependence of this parameter is determined by the classical particle trajectory, which is determined, among other things, by its initial velocity. Therefore, the subsequent quantization of motion acquires features of quasi-classical approximation. As a result, the Hamiltonian (12) acquires dependence on the initial velocity of the particle. This dependence is then transferred to the Schrödinger equation (13). In turn, this circumstance imposes restrictions on the range of possible initial wave functions, which must correspond to classical initial conditions. This is the drawback of the approach proposed here. On the other hand, a successful choice of the initial wave function of the particle can lead to physically reasonable results. Here, the well-known function of the form (16), corresponding to the coherent state of the particle, is chosen as such function. At subsequent moments of time, the wave function takes the form (17), which corresponds to a non-stationary quasi-classical coherent state.

In addition to the criteria mentioned above, the applicability condition of the quasi-classical approximation can be visually expressed in the form of inequality $l_D \gg \Delta x_{j0}$. Thus, the wave properties of the particle become noticeable at propagation distances significantly exceeding the initial coordinate uncertainties.

An important result is the statement that dissipation suppresses the quantum properties of the particle. Therefore, the dissipative medium can be considered as a classical device continuously measuring the state of a particle propagating in this medium.

Of further interest is a non-trivial generalization of the approach proposed here to the case when, in addition to internal dissipative forces, external forces of a non-dissipative nature act on the particle. This will expand the scope of consideration of dissipative media as particle detectors in their quasi-classical states.

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