JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS, 2024, Vol. 165, No. 4, pp. 596–609

STATISTICAL, NONLINEAR, AND SOFT MATTER PHYSICS

THERMODYNAMIC CRITERION OF NEUTRAL STABILITY OF SHOCK WAVES IN HYDRODYNAMICS AND ITS IMPLICATIONS

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Received October 30, 2023
Revised December 16, 2023
Accepted December 17, 2023

Abstract. It is shown that the Kontorovich criterion for neutral stability of relativistic shock waves (the relativistic analog of the Dyakov-Kontorovich criterion in classical hydrodynamics), after eliminating the derivative along the Taub-Hugoniot shock adiabat using relations at the relativistic shock-wave discontinuity, reduces to a constraint on the isenthalpic derivative of internal energy with respect to specific volume in the rest frame: $p > -\left(\partial\varepsilon/\partial v\right)_{\omega} > p_0$. The obtained formulation is also valid in classical hydrodynamics. The implications of this formulation for shock waves with single-phase and two-phase final states in a medium with first-order phase transition are derived. The influence of the Riedel parameter and isochoric heat capacity on the realizability of neutrally stable shock waves is shown. In a model problem formulation, the effect of local thermodynamic non-equilibrium on the damping of perturbations of a neutrally stable shock wave is investigated.

Keywords: *relativistic hydrodynamics, shock wave, neutral stability, phase*

DOI: 10.31857/S004445102404e138

1. INTRODUCTION

The linear stability theory of shock waves in media with arbitrary thermodynamic properties identifies a range of shock wave parameters in which, within the framework of analyzing the linearized problem for equations describing the evolution of perturbations, the latter neither grow nor decay. The energy flux of the acoustic component of secondary waves in this case is directed from the shock wave toward the shock-compressed matter, i.e., the shock wave is a source of forced sound radiation if nonlinear stability occurs, or spontaneous otherwise. To designate this range, the literature uses terms such as: Dyakov-Kontorovich instability, spontaneous sound emission by shock wave, neutral (within linear theory) stability of shock wave. Complex behavior of shock waves is associated with the fulfillment of neutral stability conditions, consisting in anomalously slow damping of perturbations and related inhomogeneity of parameters behind its front.

For the first time, the analysis of the stability of a non-relativistic shock wave to small two-dimensional perturbations in a medium with arbitrary thermodynamic properties was performed by S. P. Dyakov [1] using the normal mode method and later refined by V. M. Kontorovich [2]. Within the framework of the study, shock wave instability criteria were obtained, linking the dimensionless derivative along the shock adiabat $L = j^2(\partial V / \partial p)_H$, $j = \rho v$ is mass flux density through the shock wave surface, the Mach number of the flow behind the shock wave front (M) and the compression ratio of matter in the shock wave:

$$L < -1 \quad \text{or} \quad L > 1 + 2M \quad \text{(instability)}, \tag{1}$$

$$L > \frac{1 - M^2(1 + V_0 / V)}{1 - M^2(1 - V_0 / V)} \quad \text{(neutral instability)}. \tag{2}$$

Subsequently, these conditions were re-derived based on a more general mathematical approach [3]. In Kontorovich's work [4], the analysis was generalized to the case of relativistic hydrodynamics, and a relativistic formulation of stability criteria was obtained, which in the non-relativistic limit transitions to (1), (2). In [5], a relativistic analog of the parameter L was introduced and a formulation of shock wave stability criteria obtained in [4] was proposed through this parameter. Until now, the fulfillment of (2) has been observed for shock waves in metals [6–8], under conditions of non-equilibrium ionization in gases [9, 10], in real gas [11–13]. The fulfillment of this condition has been established in hot plasma of carbon, silicon, aluminum, niobium [14], under non-adiabatic conditions during reactions [15, 16]. In most cases, the realization of the condition was discovered as a result of direct verification of the fulfillment of (2) on the shock adiabat.

For the first time, the fact that the neutral stability condition of a shock wave, after excluding the parameter L using relations at the shock-wave discontinuity, significantly simplifies, was pointed out in [17] for the case of a non-relativistic shock wave. Later, the stability of relativistic shock waves in the linear approximation was re-examined in [18, 19] based on a method similar to [3]. The study of the well-posedness of the mixed problem for equations describing the evolution of perturbations using the Lopatinskii condition led to the formulation of the neutral stability criterion for shock waves in the form of restrictions on the derivatives of the equation of state. Based on the obtained results, the stability of shock waves for certain equations of state was considered [11, 20]. The thermodynamic formulation in the form of restrictions on thermodynamic derivatives has the advantage of a direct connection between the thermodynamic properties of the medium and shock wave stability, without constructing shock adiabats for a specific equation of state, and serves as a convenient tool for analyzing the realizability of neutrally stable shock waves in relativistic and classical hydrodynamics.

This work is devoted to studying this connection for media with phase transitions. In Section 2, based on Kontorovich's result [4], an equivalent thermodynamic formulation of the neutral stability criterion for relativistic shock waves is obtained in the form of restrictions on the isenthalpic derivative of internal energy with respect to specific volume. Section 3 provides examples of applying this thermodynamic criterion to assess the realizability of neutral stability of shock waves in media with various equations of state. Section 4 shows that in the thermodynamic formulation, the neutral stability criterion for shock waves is written identically in both relativistic and non-relativistic cases. Section 5 derives the implications of this formulation for shock waves with single-phase and two-phase final states in a medium with a first-order phase transition. The influence of the Riedel parameter and isochoric heat capacity on the realizability of neutrally stable shock waves is shown. Section 6, within the framework of a model problem based on the results of previous sections, shows the influence of non-equilibrium internal degrees of freedom of molecules on the attenuation of perturbations in a neutrally stable shock wave.

2. THERMODYNAMIC CRITERION OF NEUTRAL STABILITY OF SHOCK WAVES

The criterion of neutral stability of a relativistic shock wave within the special theory of relativity was first obtained by Kontorovich based on stability analysis using the normal modes method. The result is presented in the reference frame associated with the shock wave discontinuity, using a system of units where the speed of light equals 1. The criterion is formulated as the following chain of inequalities [4]:

$$-\frac{1}{u_y^2}\left(1+2\gamma\frac{u_y}{c}\right) < \left(\frac{\partial h}{\partial p}\right)_H < -\frac{1}{u_y^2}\frac{1-M^2-(1+2u_y^2)M^2/(u_y^2\alpha)}{1-M^2+M^2/(u_y^2\alpha)}. \quad (3)$$

Here

$$\gamma = 1/\sqrt{1-v^2},$$

$$\alpha = [h]/[p] - 2,$$

where $[\cdot]$ denotes the jump of the corresponding quantity at the shock wave discontinuity; $u_y = \gamma v$ is a component of the 4-velocity vector normal to the shock wave surface; v is corresponding component of hydrodynamic velocity, $M = v/c$ is Mach number of the flow behind the shock wave front, c is sound velocity:

$$c^2 = (\partial p/\partial e)_S;$$

e is internal energy density; p is pressure; $h = e + p$ is enthalpy density; derivative $(\partial h/\partial p)_H$ is taken along the Taub-Hugoniot shock adiabat [21]:

$$h_0^2V_0^2 - h^2V^2 + (p-p_0)(h_0V_0^2 + hV^2) = 0, \quad (4)$$

where index "0" refers to the state before the shock wave; V is specific volume in the bound reference frame. Depending on the system under consideration, specific quantities are per particle, per unit of baryon number, or per unit mass.

The left inequality in (3) for determining the boundary of neutral stability of a shock wave is not essential, since the instability region

$$-\frac{1}{u_y^2}\left(1+2\gamma\frac{u_y}{c}\right) > \left(\frac{\partial h}{\partial p}\right)_H, \tag{5}$$

corresponding to exponential growth of perturbations on the shock adiabat always overlaps with the region of structural instability or metastable behavior of the shock wave. The right inequality is of interest since it defines the boundary between the region of stable shock waves and the region of neutral stability. Let us consider the conditions that the equation of state must satisfy for shock waves meeting (3) to be possible. For this, we transform the right inequality (3), excluding the derivative along the shock adiabat and velocity, using relations at the relativistic shock wave discontinuity. Let us write this inequality in an equivalent form

$$\frac{1+u_y^2 q}{2(1+u_y^2)} < \frac{1}{1+(M^{-2}-1)u_y^2\alpha}, \tag{6}$$

where by definition

$$q = \left(\frac{\partial h}{\partial p}\right)_H,$$

and transform separately the left and right parts of (6). According to (4), the increments of energy density, pressure, and specific volume along the Taub-Hugoniot shock adiabat are related by

$$\frac{X_0}{X}dp + \frac{[p]-h}{X}dX = de, \tag{7}$$

where $X \equiv hV^2$. Using the relation at the shock wave discontinuity

$$\frac{X_0}{X} = u_y^{-2}\frac{[p]}{h} + 1, \tag{8}$$

identity

$$v^2 = \frac{u_y^2}{1+u_y^2}$$

and introducing notations

$$g = \frac{[p]}{h} - 1, \bar{V}_p = \frac{h}{V}\left(\frac{\partial V}{\partial p}\right)_e, \bar{V}_e = \frac{h}{V}\left(\frac{\partial V}{\partial e}\right)_p, \tag{9}$$

we obtain an expression for the left side of (6):

$$\frac{1+u_y^2 q}{2(1+u_y^2)} = \frac{1+g(\bar{V}_p v^2 - \bar{V}_e)}{1+g(\bar{V}_p c^2 - \bar{V}_e)}. \tag{10}$$

From the relations at the relativistic shock wave discontinuity follows the expression for parameter α:

$$\alpha = \left(1-\frac{[p]}{h}\right) \Big/ \left(u_y^2 + \frac{[p]}{h}\right). \tag{11}$$

Taking into account (11), the right side of (6) reduces to

$$\frac{1}{1+(M^{-2}-1)u_y^2\alpha} = \frac{1+g(1-v^2)}{1+g(1-c^2)}. \tag{12}$$

After substituting the transformed expressions (10) and (12) into (6), we have

$$\frac{1+g(\bar{V}_p v^2 - \bar{V}_e)}{1+g(\bar{V}_p c^2 - \bar{V}_e)} < \frac{1+g(1-v^2)}{1+g(1-c^2)}. \tag{13}$$

From the relation for the square of sound speed

$$c^2 = -(1+\bar{V}_e)/\bar{V}_p > 0,$$

the constraint on parameter g, which follows from the relations at the shock wave discontinuity:

$$-1 < g < -1/2,$$

the causality principle $c < 1$ and the shock wave discontinuity evolution condition $v < c$ it follows that the denominator of the right side of (13) is positive, and the sign of the denominator of the left side of the inequality is opposite to the sign of $\bar{V}_p$. After reducing to a common denominator, dividing by g, $\bar{V}_p$ and $v^2 - c^2$, we arrive at an inequality equivalent to (13),

$$[p]\left(1-\bar{V}_e/\bar{V}_p\right) > hc^2. \tag{14}$$

Taking into account the identity

$$1-\bar{V}_e/\bar{V}_p = w_\varepsilon\big|_V,$$

where

$$w_\varepsilon\big|_V = \left(\frac{\partial w}{\partial \varepsilon}\right)_V,$$

$\varepsilon = Ve$ is internal energy, $w = \varepsilon + pV$ is enthalpy, (14) takes the form

$$hc^2 - w_\varepsilon\big|_V [p] < 0. \tag{15}$$

Condition (15) is equivalent to Kontorovich's criterion of neutral stability of a shock wave. The thermodynamic identity

$$w_\varepsilon\big|_V(\varepsilon_V\big|_w + p) = hc^2,$$

proof of which is given in the Appendix, can be written as:

$$w_\varepsilon\big|_V(G + [p]) = hc^2, \tag{16}$$

where

$$G = \varepsilon_V\big|_w + p_0.$$

Expressing the derivative $w_\varepsilon\big|_V$ from (16), after substitution into (15) we obtain an equivalent form of the shock wave neutral stability condition:

$$\frac{G}{G + [p]} hc^2 < 0, \tag{17}$$

from which it follows that the neutral stability condition is satisfied if and only if the parameter G is within the range $-[p] < G < 0$, which is equivalent to the constraint on the internal energy derivative:

$$p > -\varepsilon_V\big|_w > p_0. \tag{18}$$

The left inequality in (18) is equivalent to the condition $w_\varepsilon\big|_V > 0$, another form of which is $\Gamma > -1$, where

$$\Gamma = V\, p_T\big|_V / \varepsilon_T\big|_V$$

is the Gruneisen parameter. Violation of this condition seems exotic, although it does not contradict the laws of thermodynamics. Under these conditions, the right inequality acquires the force of a criterion

$$-\varepsilon_V\big|_w > p_0, \tag{19}$$

which is most often not satisfied, and its fulfillment in a limited region of the phase diagram means the realizability of neutral stability of shock waves. This condition is satisfied with a negative derivative in the left part of (19), primarily for high-intensity shock waves. Let us provide several examples.

3. EXAMPLES OF APPLYING THE THERMODYNAMIC CRITERION

The relativistic equation of state for a gas of non-interacting particles [22],

$$w = \frac{K_3(1/(w-\varepsilon))}{K_2(1/(w-\varepsilon))}, \tag{20}$$

where K_2 and K_3 are modified Bessel functions of the second kind of second and third order, does not allow the fulfillment of (19) since the left side of the inequality equals zero. Similarly, the equations of state for ultra-relativistic gas, radiation, non-relativistic ideal gas, as well as any caloric equation of state connecting enthalpy and internal energy through a functional dependence of the form $f(w,\varepsilon) = 0$, or in parametric notation $pV = f(T)$, $\varepsilon = \varepsilon(T)$, allow for the existence of only stable shock waves within the framework of linear theory [1–4]. One of the applications of relativistic hydrodynamics is modeling shock compression of nuclear matter during the collision of relativistic nuclei in colliders, leading to the formation of quark-gluon plasma and its subsequent expansion and hadronization. During the collision stage, the parameters of quark-gluon plasma are estimated from the relations at the shock wave discontinuity, and the question of shock wave stability has been raised in literature, see, for example, [23–25]. The caloric equation of state for quark-gluon plasma within the framework of the M.I.T. bag model (see [26]), which neglects quark masses during its derivation, has the form

$$w = \frac{4}{3}(\varepsilon - BV), \tag{21}$$

where $B > 0$ is the bag model constant. Consequently, the fulfillment of (3) for shock waves with a final state in the region of the nuclear matter phase diagram corresponding to quark-gluon plasma is impossible if the influence of corrective amendments to (21) does not exceed the stabilizing influence of the constant B.

Let the equation of state of the substance be given in parametric form

$$p = p(V,T), \quad \varepsilon = \varepsilon(V,T). \tag{22}$$

Let us transition in (19) from variables (p,w) to variables (V,T). Such transition is one-to-one. As a result, we obtain

$$(pV)_V|_T - \xi(Vp_V|_T + Tp_T|_V) > p_0, \tag{23}$$

$$\xi = \frac{1}{1 + \varepsilon_T|_V/(Vp_T|_V)}.$$

The necessary condition for the realizability of neutral stability, corresponding to the limit of shock waves of infinite intensity, ($p_0 \to 0$) in (23), can be written as

$$(pV)_V|_T\, c_V > Vp_T\big|_V\, \varepsilon_V\big|_T, \tag{24}$$

where

$$\varepsilon_V\big|_T = Tp_T|_V - p, \quad c_V = \varepsilon_T\big|_V.$$

Consequently, for a medium with a positive Gruneisen parameter, which is the most common case for real media, the independence of internal energy from volume $\varepsilon = \varepsilon(T)$ or the negativity of the derivative $\varepsilon_V\big|_T < 0$ at $(pV)_V|_T > 0$ means unconditional (independent of ñ$_V$) realizability of neutrally stable shock waves. In this case, there exists a threshold intensity of the shock wave above which the Dyakov-Kontorovich criterion is satisfied. In the remaining cases, the realizability of neutral stability is determined by the magnitude of isochoric heat capacity. Conversely, if $(pV)_V|_T < 0$, the non-negativity of $\varepsilon_V\big|_T$ implies unconditional stability of shock waves according to this criterion. In such examples, we see how the property of forced or spontaneous sound emission by a shock wave simultaneously with the fact of its neutral stability within the framework of linear theory is determined from the equation of state without constructing shock adiabats and checking the criterion in its original form.

4. NON-RELATIVISTIC LIMIT

The form of the neutral stability condition for shock waves (18) obtained within the framework of a more general theory is equally valid for relativistic and non-relativistic shock waves. To illustrate this statement, let's derive it directly from (2). The increments of variables along the shock adiabat

$$\varepsilon - \varepsilon_0 + \tfrac{1}{2}(p + p_0)(V - V_0) = 0 \tag{25}$$

are related by the equation

$$\varepsilon_V|_p\, dV + \varepsilon_p|_V\, dp + \frac{V - V_0}{2}dp + \frac{p + p_0}{2}dV = 0, \tag{26}$$

which, taking into account the expression for sound speed and relations at the shock wave discontinuity, leads to the following expression for the Dyakov parameter:

$$L = -1 + \frac{1 - M^2}{1 - \frac{1}{2}(p - p_0)/(p + \varepsilon_V|_p)}. \tag{27}$$

Since, on the other hand, (2) is equivalent to

$$L > -1 + \frac{1 - M^2}{\frac{1}{2}\left(1 - M^2(V - V_0)/V\right)}, \tag{28}$$

the Dyakov-Kontorovich condition can be written as

$$\frac{1 - M^2}{1 - \frac{1}{2}(p - p_0)/(p + \varepsilon_V|_p)} > \frac{1 - M^2}{\frac{1}{2}\left(1 - M^2(V - V_0)/V\right)}. \tag{29}$$

$M < 1, V_0 > V$, consequently, both fractions are positive and the condition takes the form

$$M^2\frac{V_0 - V}{V} > 1 - \frac{p - p_0}{p + \varepsilon_V|_p}. \tag{30}$$

Taking into account the identities

$$\frac{c^2}{V^2} = \frac{p + \varepsilon_V|_p}{\varepsilon_p|_V}, \quad M^2 = \frac{p - p_0}{V_0 - V}\frac{V^2}{c^2} \tag{31}$$

we have

$$w_\varepsilon\big|_V[p] > c^2/V, \tag{32}$$

which is the non-relativistic analogue of (15). Using the non-relativistic limit of identity (16)

$$w_\varepsilon\big|_V(G + [p]) = c^2/V, \tag{33}$$

we finally have

$$p > -\varepsilon_V|_w > p_0.$$

As expected, we arrive at the result obtained within the framework of relativistic hydrodynamics. The normal modes method and the method of investigating the correctness of the mixed problem for perturbations equally define the stability boundaries; accordingly, the constraints on the derivatives of the equation of state behind the front of a neutrally stable shock wave [11]

$$\frac{\rho}{p}\left(\frac{\partial p}{\partial \rho}\right)_\varepsilon < 1, \quad 1 + \frac{1}{\rho\varepsilon_p|_V} > 0 \tag{34}$$

are equivalent to (18). Indeed, the relations

$$-\varepsilon_V\,|_w = \frac{w_V|_\varepsilon}{w_\varepsilon|_V} = \frac{p + V\,p_V|_\varepsilon}{1 + V\,/\,\varepsilon_p|_V} > p_0,$$

$$p + \varepsilon_V\,|_w = \frac{hc^2}{w_\varepsilon|_V} = \frac{hc^2}{1 + V\,/\,\varepsilon_p|_V} > 0$$

show the connection between (34) and (18). Moreover, (18) has a simple thermodynamic interpretation: neutrally stable shock waves are possible only under conditions where internal energy decreases during expansion in an isenthalpic process, and the boundaries of neutral stability in the space of thermodynamic variables are the level lines of the derivative of internal energy with respect to volume $\varepsilon_V|_w$.

5. NEUTRAL STABILITY OF SHOCK WAVES AND FIRST-ORDER PHASE TRANSITION

5.1. Two-phase states behind the shock wave front

For shock waves with final state in the two-phase region of phase transition ($p = p_s(T)$ and $p_V\,|_T = 0$, where p_s is the pressure on the saturation line) condition (24) takes the form

$$\frac{c_V}{R} > \theta\left(\theta\,/\,(1 - I^{-1}) - 1\right)Z, \tag{35}$$

where

$$Z = \frac{pV}{RT}$$

is the compressibility coefficient,

$$\theta = \frac{d(\ln p_s)}{d(\ln T)}$$

is a temperature function characterizing the slope of the phase equilibrium curve in the plane of variables (p,T), $I = p\,/\,p_0$ is the pressure drop at the shock wave front. The necessary condition for the realizability of neutrally stable shock waves, corresponding to the limit of shock waves of infinite intensity, is

$$\frac{c_V}{R} > \theta\left(\theta - 1\right)Z. \tag{36}$$

From (36) follows unconditional (regardless of c_V) realizability of neutral stability of strong shock waves at $0 < \theta < 1$. The shock wave in this case is neutrally stable at pressure drop on the front $I > (1 - \theta)^{-1}$. In the case of $\theta < 0$ and $\theta > 1$ the realizability of such shock waves is determined by the value of isochoric heat capacity. The liquid-gas phase transition for a wide range of substances corresponds to the case $\theta > 1$.

The value θ at the critical point is a similarity parameter of thermodynamic properties of various substances (Riedel parameter, $\alpha = (d(\ln p)\,/\,d(\ln T))_c$) and within the law of corresponding states is approximated by the dependence

$$\alpha = 4.919\omega + 5.811,$$

where ω is Pitzer's acentric factor, which, taking into account the correlation for compressibility

$$Z_c = 0.291 - 0.080\omega$$

gives

$$\left(\theta\left(\theta - 1\right)Z\right)_c \approx 8.135 + 12.97\omega.$$

At the boundary of the two-phase region from the side of saturated liquid and saturated vapor, the right part of (36) is a function of temperature. These dependencies, following from the law of corresponding states for the acentric factor of water $\omega = 0.344$ [27], are shown in Fig. 1, which also presents data on the heat capacity of liquid water and vapor at the boundary of the two-phase region from the side of two-phase states [28]. The right and left parts of (36) within the two-phase region

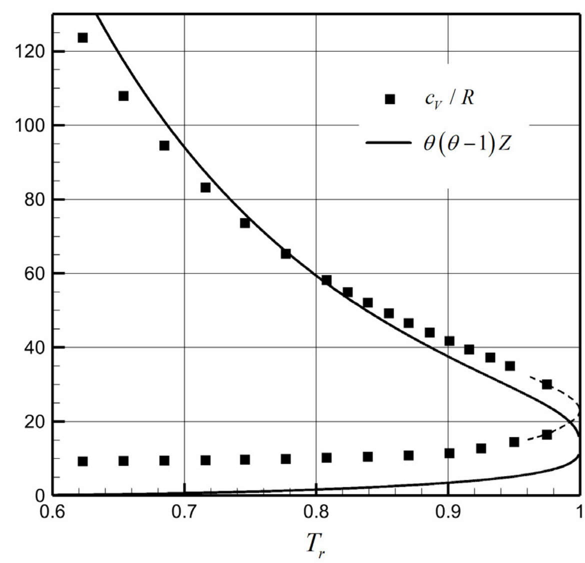

Fig. 1. Left and right parts of (36) for $H_2 0$ at the boundary of the two-phase region depending on the reduced temperature

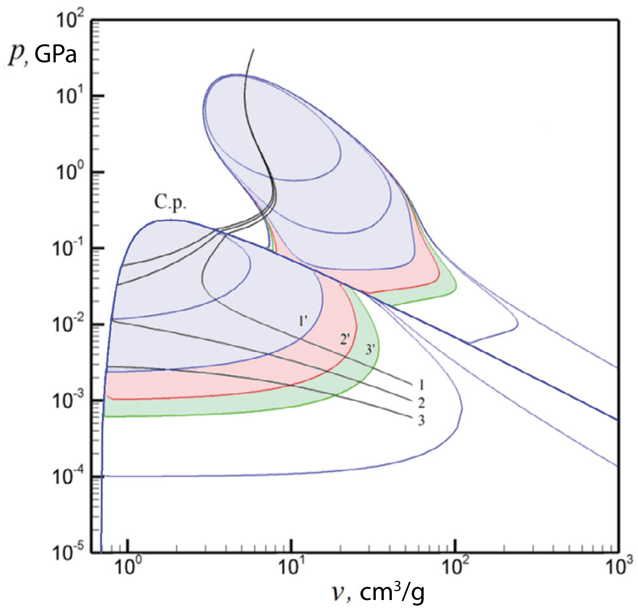

Fig. 2. Level lines $\varepsilon_V|_w$ for magnesium, k' is the boundary of neutral stability $\partial\Omega^{(p_k)}$, where p_k is initial pressure of shock adiabat k, C.p. is critical point

on an isotherm are linear functions of specific volume; therefore, fulfillment of the inequality at the boundaries of the two-phase region from the liquid and gas sides is sufficient for it to be fulfilled at all internal points. From Fig. 1, it follows that (36) is satisfied in the near-critical region at temperatures exceeding approximately 0.78, at all values of specific volume. At lower reduced temperatures, two-phase states with a predominant vapor fraction appear, for which (36) is not satisfied. A similar pattern occurs for other substances. In [7], based on a wide-range equation of state, a similar pattern of neutral stability of shock waves with final state in the two-phase region of metals is shown. In Fig. 2 for the wide-range equation of state of magnesium (see [7]), regions of neutral stability of shock waves are shown as level lines $\varepsilon_V|_w$. The designation k' corresponds to the boundary of region $\partial\Omega^{(p_k)}$, where p_k is initial pressure of shock adiabat k. Each of the shown shock adiabats has sections of neutral stability at the intersection with the corresponding region $\Omega^{(p_k)}$ both in the two-phase and single-phase regions.

5.2. Single-phase states behind the shock wave front

Let us consider the consequences of (23) for shock waves with a final state in the single-phase region of the phase transition with a positive slope of the phase equilibrium curve at the critical point $\theta_c > 0$. From continuity $p_V|_T$ during the transition through the critical point from the two-phase region to the single-phase region, it follows that between the binodal of the phase transition and the Boyle curve (defined by the condition $(pV)_V|_T = 0$) in variables (V, p) holds $(pV)_V|_T > 0$. From the continuity of the derivative $p_T|_V$ at the critical point (Planck-Gibbs relation $(p_T|_V)_c = (dp_s / dT)_c$, meaning that at the critical point the slope of the saturation line in coordinates (T, p) equals the slope of the critical isochore) follows the continuity of $\varepsilon_V|_T$, while from the identity

$$(\partial(\varepsilon_V|_T) / \partial T)|_V = (\partial c_V / \partial V)|_T$$

follows the boundedness of its temperature derivative. Note that models in which the specific heat at constant volume depends only on temperature, lead to the constancy of $\varepsilon_V|_T$ on the isochore. Consequently, if the slope of the phase equilibrium curve in the plane of variables (T, p) is within the interval $0 < \theta_c < 1$, in the vicinity of the critical point from the single-phase states side there exists an intersection of regions $(pV)_V|_T > 0$ and $\varepsilon_V|_T < 0$, corresponding to unconditional realizability (independent of specific heat c_V) of neutral stability of shock waves.

The neutral stability boundary on the (V, p) diagram in this case is located between the Boyle curve and curve $\varepsilon_V|_T = 0$ and passes through their intersection points, if any exist. If the slope of the phase equilibrium curve satisfies condition $\theta_c > 1$, then in the vicinity of the critical point from the side of single-phase states, conditions $(pV)_V|_T > 0$ and $\varepsilon_V|_T > 0$ are satisfied. The liquid-gas phase transition corresponds to this case. Let $\varepsilon_V|_T$ maintain its sign throughout the entire region above the binodal, as is the case for the real gas models considered below. Then, depending on the magnitude of isochoric heat capacity, neutral stability of shock waves is possible only for states behind the shock wave front enclosed between the binodal and the Boyle curve. Since the limit of high heat capacity corresponds to the fulfillment of the neutral stability condition for shock waves for all states between the binodal and the Boyle curve, and the limit of low heat capacity corresponds to its non-fulfillment (in this limit (23) takes the form $-\varepsilon_V\big|_T > 0$), there exists a threshold value of heat capacity at which neutrally stable shock waves become possible. Let us estimate this threshold based on the thermodynamic criterion of neutral stability of shock waves for some real gas models.

5.3. Influence of Heat Capacity

Let the state of liquid and gas be described by a single equation of state $\varepsilon = \varepsilon(p,V)$, as in the case of the Van der Waals equation. From (29) we conclude that the boundary $\partial\Omega^{(p_0)}$ of the neutral stability region passes through the intersection points of the straight line $p = p_0$ and the hyperbolicity boundary, defined by condition $c = 0$, where c is the adiabatic sound speed. These points are located on the plane (V,p) below the spinodal and correspond to thermodynamically unstable states. Let us consider the equation of state of general form:

$$p = r(V)RT - A(V), \tag{37}$$

where $r(V)$, $A(V)$ are specific volume functions. Dependence (37) generalizes the Van der Waals gas equation of state, the second Dieterici equation, various approximations of the hard spheres model, refining the function $r(V)$, and attraction term models $A(V)$. To evaluate the feasibility of neutral stability of shock waves in a single-phase region, the weak temperature dependence of this term, which is considered in semi-empirical equations of state, can be linearized near the binodal. With a known temperature dependence of isochoric heat capacity of the form $\varepsilon_T\,|_V = c_V(T)$ the fundamental equation of such gas is written in parametric form:

$$\begin{aligned} \varepsilon(V,T) &= \varepsilon_0 + \int_0^T c_V(t')dt' + \int_\infty^V A(v')dv', \\ s(V,T) &= s_0 + \int_0^T \frac{c_V(t')}{t'}dt' + R\int_\infty^V r(v')dv'. \end{aligned} \tag{38}$$

The square of adiabatic sound speed in such medium is

$$c^2 = V^2\left(\left(\frac{rR}{c_V} - \frac{r'}{r}\right)(p + A) + A'\right), \tag{39}$$

$$r' = dr\,/\,dV, \quad A' = dA\,/\,dV.$$

Using equation (39), we express the square of the speed of sound through pressure at $c = 0$ representing it as a function of volume $p\big|_{c=0} = \mathcal{H}(V,c_V)$:

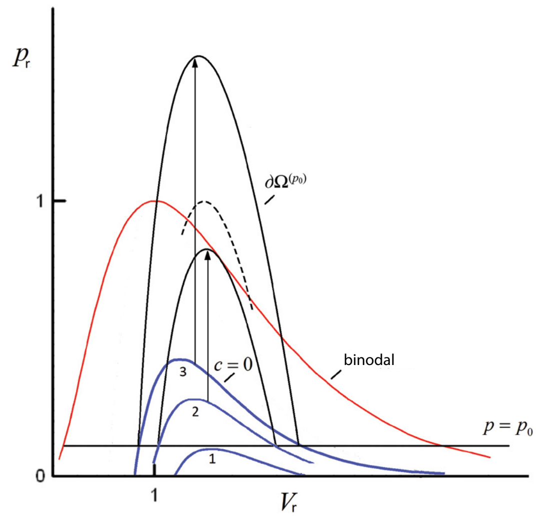

Fig. 3. Diagram showing the dependence of the neutral stability region $\partial\Omega^{(p_0)}$ in real gases on isochoric heat capacity. Curves 1–3 are adiabatic spinodal for three values of heat capacity $c_{V3} > c_{V2} > c_{V1}$. $\Omega^{(p_0)}$ for each value c_V is determined by inequality (43)

$$c^2 = V^2\left(\frac{rR}{c_V} - \frac{r'}{r}\right)(p - \mathcal{H}(V,c_V)), \tag{40}$$

$$\mathcal{H}(V,c_V) = -A'\,/\left(\frac{rR}{c_V} - \frac{r'}{r}\right) - A. \tag{41}$$

For the given medium:

$$e_p\big|_V = \frac{1}{V}\frac{\varepsilon_T\big|_V}{p_T\big|_V} = \frac{1}{rV}\frac{c_V}{R}. \tag{42}$$

Substituting equations (40) and (42) into equation (32) leads to the condition for neutral stability of the shock wave:

$$p - p_0 < \lambda(\mathcal{H}(V,c_V) - p_0), \tag{43}$$

$$\lambda = V\left(\frac{r'}{r} - \frac{R}{\tilde{n}_V}r\right)\Big/\left(1 + V\frac{r'}{r}\right). \tag{44}$$

In the case of a gas with constant isochoric heat capacity, the curve $p = \mathcal{H}(V,c_V)$ represents the adiabatic spinodal. The boundary of the neutral stability region $\partial\Omega^{(p0)}$ in the (V, p) plane, according to equation (43), is the image of the adiabatic spinodal stretched relative to the line $p=p_0$ along the p-axis with a coefficient that depends on volume and heat capacity. This provides a simple qualitative picture of the neutral stability region relative to the binodal, as shown in Fig. 3. It follows that if p_0 exceeds the maximum pressure on the adiabatic spinodal (curve 1), equation (18) is not satisfied even in the metastable and unstable states.

With an increase in heat capacity, the adiabatic spinodal shifts to the region of higher pressures, and at a certain value of heat capacity, $\partial\Omega^{(p0)}$ enters the region of thermodynamically stable single-phase states (curve 2). With further increases in heat capacity, supercritical pressures behind the front of the neutrally stable shock wave become possible (shown by the dashed line). At even higher heat capacities, $\partial\Omega^{(p0)}$ passes through the critical point of the phase transition, and neutrally stable shock waves with supercritical density behind the shock front become possible (curve 3).To demonstrate the sensitivity of threshold heat capacity values to the parameter θ_c,which according to equation (36) plays an important role in the feasibility of neutrally stable shock waves in the near-critical region, we provide quantitative estimates of threshold heat capacity values for Van der Waals and Dieterici gases. According to Van der Waals equation, the Riedel similarity parameter $\theta_c = 4$, while for Dieterici equation $\theta_c = 5$, which is significantly closer to experimental values. For Van der Waals and Dieterici gases

$$r(V) = (V - b)^{-1}, \quad A(V) = \frac{a}{V^n},$$

and condition (43) has a simple form

$$\begin{gathered} p - p_0 < \gamma \frac{V}{b}(\mathcal{H}(V, c_V) - p_0), \\ \mathcal{H}(V, c_V) = \frac{a}{\gamma V^n}(n - \gamma - nb / V), \end{gathered} \tag{45}$$

where

$$\gamma = 1 + R / c_V.$$

According to (45), the maximum pressure at the neutral stability boundary is

$$\max_{\partial\Omega^{(0)}}(p) = \frac{a}{b^n}\frac{(n-\gamma)^n(n-1)^{n-1}}{n^{2n-1}}. \tag{46}$$

Taking into account critical point parameters

$$V_c = \frac{n+1}{n-1}b, \quad p_c = \frac{a}{b^n}\left(\frac{n-1}{n+1}\right)^{n+1}, \quad \theta_c = \frac{2n}{n-1},$$
$$\frac{RT_c}{p_cV_c} = \frac{4n}{(n-1)(n+1)}$$

the pressure maximum at $\partial\Omega^{(0)}$, relative to the pressure at the critical point, is

$$\max_{\partial\Omega^{(0)}}(p / p_c) = \frac{(n-\gamma)^n(n+1)^{n+1}}{(n-1)^2 n^{2n-1}}. \tag{47}$$

According to (47), the neutral stability boundary of shock waves extends into the supercritical pressure region under the condition

$$\gamma < n - \frac{(n-1)^{2/n} n^{(2n-1)/n}}{(n+1)^{(n+1)/n}}, \tag{48}$$

which gives

$$\gamma < 2 - (2/3)^{3/2} \approx 1.455$$

for Van der Waals gas ($n = 2$) and

$$\gamma < 5/3(1 - 5^{2/5} / 2^{18/5}) \approx 1.405$$

for Dieterici gas ($n = 5/3$).

The condition for achieving supercritical densities in a neutrally stable shock wave is obtained by substituting critical point parameters into (45)

$$\gamma < \left(\frac{2n}{n-1} - \frac{n-1}{n+1}(1 - p_0)\right) \Big/ \left(p_0 + \frac{n+1}{n-1}\right). \tag{49}$$

For Van der Waals and Dieterici equations of state, we have $\gamma < 1.222$ and $\gamma < 1.187$ respectively. It should be noted that these values are close to the feasibility condition for neutrally stable shock waves with initial state in the single-phase region. According to [13], the realization of such shock waves becomes possible at $\gamma \approx 1.215$ and $\gamma \approx 1.199$ respectively. This condition corresponds to the tangency of the neutral stability region boundary $\partial\Omega^{(p_0)}$ with shock adiabat having initial point on the phase transition binodal at pressure p_0.

The obtained estimates are consistent with the general trend derived from (24) and continuity θ during the transition through the critical point: the higher the Riedel similarity parameter for the phase transition, the higher the threshold values of isochoric heat capacity at which neutrally stable shock waves are realized. Since the experimental values of the Riedel parameter ($\theta_c \approx 5.8$) exceed the values for the Dieterici equation ($\theta_c = 5$) and the Van der Waals equation, ($\theta_c = 4$), even more stringent restrictions can be expected for real substances than these models predict. In this case, the heat capacities of translational and rotational degrees of freedom of molecules are insufficient for the realization of neutrally stable shock waves with a final state in the single-phase region. The presence of thermodynamic factors associated with the excitation of internal degrees of freedom is necessary, which would lead to an increase in heat capacity or other thermodynamic factors (corrections for non-ideality), resulting in a decrease in the isenthalpic derivative of internal energy with respect to specific volume.

Models of media, in which until now several authors have noted the fulfillment of the neutral stability condition for shock waves, provide insight into such factors. For neutrally stable shock waves with a final state in the two-phase region of the phase diagram, such a factor is phase transformations. And here we encounter the following problem.

Linear stability theory of shock waves using the method of normal modes or within the framework of studying the well-posedness of the mixed problem for perturbations considers a shock wave as a discontinuity surface behind which local thermodynamic equilibrium conditions are satisfied. At the same time, factors that lead to the fulfillment of the neutral stability condition, such as excitation of internal molecular degrees of freedom, phase transitions in multiphase media, ionization with the establishment of equilibrium between electronic and ionic subsystems, etc., suggest that an extended relaxation zone towards thermodynamic equilibrium adjoins the narrow gradient zone with predominantly viscous structure, which can be considered as a shock wave discontinuity. In this case, the neutral stability condition is not satisfied at the viscous jump in the approximation of frozen relaxation processes. It is expected that the interaction of the shock wave discontinuity with the relaxation zone will lead to those properties of long-wave two-dimensional perturbations that linear theory predicts, namely: a change in the decay law of shock wave perturbations compared to the case when the shock wave is stable in linear approximation; forced (or spontaneous) sound emission by the shock wave. Here we refer to the results of recent works [29, 30], in which for a shock wave satisfying condition (2), a linear stability analysis was performed taking into account the relaxation structure, and it was shown that the interaction of the shock wave and the adjacent relaxation zone is consistent with the classical theory conclusion about sound emission by the shock wave.

However, linear analysis does not allow determining the fact of stability or instability of the shock wave when the Dyakov-Kontorovich condition is satisfied and determining the decay law (or growth) of perturbations, which in this case is determined by nonlinear terms in the perturbation amplitude expansion. In fact, the fulfillment of this condition simply signals a change in the perturbation decay law compared to a shock wave that is stable within linear theory. Therefore, in the next section, we will consider the influence of non-equilibrium internal degrees of freedom on the perturbation decay rate within the framework of the nonlinear problem.

6. INFLUENCE OF THERMODYNAMIC NONEQUILIBRIUM

As a simple model of a neutrally stable shock wave, a shock wave with a final state in a single- phase near-critical region of the liquid-gas phase transition in a gas with the equation of state is considered

$$p(\rho,T) = \frac{\rho RT}{1-b\rho} - a\rho^2,$$

$$\varepsilon(\rho,T) = c_V^0 T - a\rho + N\frac{R\Theta}{e^{\Theta/T}-1}, \qquad (50)$$

where the characteristic temperature Θ is the same for N harmonic oscillators per particle. It is assumed that when the temperature changes, the system reaches equilibrium within a characteristic time τ. The model kinetics was postulated in the form

$$\tau d\Upsilon / dt = \Upsilon^{eq} - \Upsilon,$$

where

$$\Upsilon^{-1} = c_V^0 / R + N\frac{\Theta / T}{e^{\Theta/T}-1}. \qquad (51)$$

The system describing the flow of such gas, reduced to dimensionless form using the parameters of the phase transition critical point, is

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0,$$

$$\frac{\partial\rho\mathbf{v}}{\partial t} + \nabla\cdot(\rho\mathbf{v}\mathbf{v} + \mathbf{I}p) = 0,$$

$$\frac{\partial(e+\frac{1}{2}\rho\mathbf{v}^2)}{\partial t} + \nabla\cdot\left(\left(e+\frac{1}{2}\rho\mathbf{v}^2+p\right)\mathbf{v}\right) = 0,$$

$$p = \frac{\Upsilon(e+3\rho^2)}{1-\rho/3} - 3\rho^2, \qquad (52)$$

$$\frac{\partial\rho\Upsilon}{\partial t} + \nabla\cdot(\rho\mathbf{v}\Upsilon) = \rho\tau^{-1}(\Upsilon^{eq}(p,\rho) - \Upsilon),$$

$$\Upsilon^{eq}(p,\rho) = \left(c_V^0 / R + \frac{N}{x(e^{1/x}-1)}\right)^{-1},$$

$$x = \frac{\left(p+3\rho^2\right)(\rho^{-1}-1/3)}{(3/8)\Theta},$$

where $e = \rho\varepsilon$ is the internal energy density, $\mathbf{v}$ is the velocity vector. For this system, the problem of the evolution of an initial periodic perturbation of the neutrally stable shock wave front is considered.

The flow is considered in the spatial domain

$$(x, y) \in [-l, L] \times [0, \Lambda]$$

in a reference frame where the unperturbed shock wave is stationary. The initial perturbation is given by the curvature of the shock wave discontinuity shape

$$x = f(y),$$

where

$$f(y) = (1/5)\Lambda\cos(\pi y/\Lambda),$$

Λ is the half-period of perturbation. Initial data corresponding to the neutrally stable shock wave:

$$(p, V) = (0.1, 20) \quad \text{at} \quad x < f(y);$$

$$p = 1.2 \quad \text{at} \quad x > f(y).$$

The remaining parameters were determined from the relations at the unperturbed shock wave discontinuity under conditions of equilibrium of internal degrees of freedom $\Upsilon = \Upsilon^{eq}$.

At the boundaries $y = 0$ and $y = \Lambda$ symmetry conditions are set. The condition on the boundary section $x = -l$ fixes the flow parameters before the shock wave, at the remote boundary at $x = L$ non-reflecting boundary conditions were set.

The following model parameters were selected to ensure the neutral stability condition of the shock wave at given initial state parameters and final pressure:

$$c_V^0 / R = 3/2, \quad N = 12, \quad \Theta = 3.$$

The attenuation of shock wave perturbations is determined by the time dependence of averaged pressure fluctuations on the contour C behind its front. The averaging contour is located in the relaxation zone behind the shock wave discontinuity with a constant offset relative to its current position, see Fig. 4. Fig. 5 shows the calculation results for three values of the relaxation zone half-width Δ, defined as the distance from the shock wave discontinuity

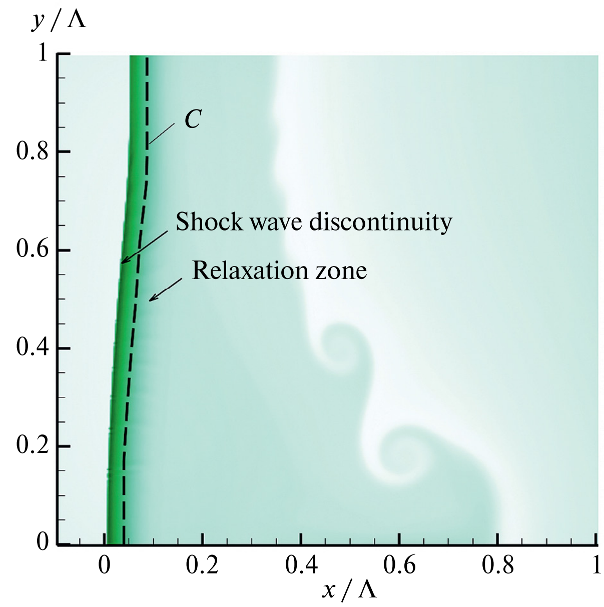

Fig. 4. Position of contour C, on which root-mean-square pressure fluctuations behind the shock wave front are calculated

at which the difference $\Upsilon - \Upsilon^{eq}$ decreases by half compared to the maximum value. The time scale t_Λ is the time during which the shock wave travels a distance equal to half the spatial period of the perturbation Λ. The presented calculations show the change in the nature of perturbation attenuation depending on the ratio between the relaxation zone width and the spatial period of the perturbation. At $\Delta/\Lambda = 0.3$ the attenuation law is observed to be close to exponential, characteristic of stable shock waves. At $\Delta/\Lambda = 0.005$ the attenuation law changes to a weaker one, which can be approximated by a power-law dependence.

The change in the attenuation law compared to a stable shock wave is quite expected for shock waves that are neutrally stable within the linear analysis framework. The stabilizing effect of the finite relaxation zone width is more pronounced for short-wave perturbations. It should be noted that this influence is quite strong: even at a ratio of perturbation wavelength to characteristic relaxation zone width of about 20, we see a significant change in the rate of perturbation attenuation.

From the calculations presented in Fig. 5, it can be concluded that regarding the rate of perturbation decay the range of neutral stability of shock waves remains distinct when accounting for the non-equilibrium structure of the shock wave. The practical significance will be determined by the

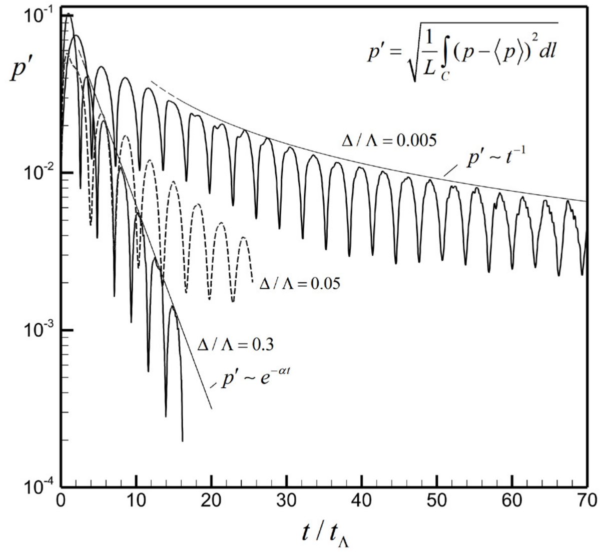

Fig. 5. Influence of relaxation zone on perturbation decay of neutrally stable shock wave; Λ is perturbation half-period, Δ is relaxation zone half-width, t_Λ is time during which shock wave travels distance Λ

width of the relaxation zone and the spectrum of perturbations in a specific problem.

Consideration of the shock wave structure influence, as known, corrects the conclusions of linear stability theory of shock waves, which considers the shock wave as a discontinuity surface. In particular, in the parameter range (1), where linear theory predicts the development of two-dimensional instability, viscous shock waves are not realized, and substance compression occurs in a combined wave [31–35]. In a medium with phase transition, such a combined compression wave can have a two-wave structure, where the precursor corresponds to substance compression in the initial phase, and the following shock wave is a phase transformation wave. A theoretical example is given by shock compression of nuclear matter under conditions of quark-hadron phase transition [25]. From the thermodynamic formulation of the neutral stability criterion (18), it follows that during the decay of a neutrally stable shock wave due to structural instability, the closing shock wave in the combined compression wave retains the property of neutral stability if the precursor intensity does not exceed the threshold value

$$\delta p < -(\varepsilon_V\big|_w + p_0).$$

In this respect, the influence of the structural factor is also conditional.

7. CONCLUSION

The thermodynamic formulation of the neutral stability criterion for shock waves coincides for relativistic and non-relativistic shock waves and reduces to a simple condition on the derivative of internal energy with respect to specific volume at constant enthalpy: $p > -\varepsilon_V\big|_w > p_0$, where index "0" corresponds to the initial state. This formulation of the criterion allows considering the feasibility of neutrally stable shock waves in media with different thermodynamic properties separately from the analysis of shock adiabats. In particular, the slope of the phase equilibrium curve in the plane (p,T) has a determining influence on the feasibility of neutrally stable shock waves in a medium with a first-order phase transition. At Riedel parameter values $\theta_c = d(\ln p_s)/d(\ln T)$, characteristic for the liquid- gas phase transition, fulfilling the neutral stability condition for shock waves with final state in the single-phase region requires high medium heat capacity exceeding that of an ideal gas considering rotational and translational molecular degrees of freedom. Results for the model equation of state, generalizing van der Waals and Dieterici equations of state, showed that as the Riedel parameter increases, the threshold value of heat capacity at which neutral stability of shock waves becomes possible also increases. High isochoric heat capacity due to the heat of phase transition contributes to fulfilling the neutral stability condition for shock waves with final state in the two-phase region of the liquid-gas phase transition. Consideration of thermodynamic factors leading to the fulfillment of the shock wave neutral stability condition, based on the thermodynamic criterion and literature data on cases of its fulfillment, indicates that the neutral stability condition is met due to the influence of the medium relaxation zone to local thermodynamic equilibrium behind the shock wave front, since a viscous jump with excitation of translational and rotational degrees of freedom does not satisfy this condition. In the non-equilibrium zone, processes occur that reduce the magnitude of the isenthalpic derivative of internal energy with respect to specific volume. Based on a simple model of molecular internal degrees of freedom relaxation for a shock wave with final state in the near-critical region of the liquid-gas phase transition, the influence of the relaxation zone on the damping rate of shock wave perturbations is shown.

FUNDING

This work was supported by the Ministry of Science and Higher Education of the Russian Federation (state assignment No. 075-01129-23-00).

APPENDIX

We transform the right side of the thermodynamic identity

$$w_\varepsilon\big|_V \, \varepsilon_V\big|_w = -w_V\big|_\varepsilon \tag{53}$$

as follows:

$$\begin{aligned} -w_V\big|_\varepsilon &= -p - V\, p_V\big|_\varepsilon = \\ &= -p - V\, p_V\big|_S - V\, p_S\big|_V\, S_V\big|_\varepsilon = \\ &= -p - V\, p_V\big|_S - V\frac{p_T\big|_V}{S_T\big|_V} S_V\big|_\varepsilon = \\ &= -p - Vp_V\big|_S - V\frac{p_T\big|_V}{\varepsilon_T\big|_V} p = hc^2 - p\, w_\varepsilon\big|_V. \end{aligned}$$

After substitution into (53) and regrouping

$$w_\varepsilon\big|_V (\varepsilon_V\big|_w + p) = hc^2, \tag{54}$$

where

$$w_\varepsilon\big|_V = 1 + \Gamma = 1 - \bar{V}_e / \bar{V}_p,$$

$\Gamma = V\, p_\varepsilon\big|_V$ is Gruneisen parameter, definition of dimensionless parameters $\bar{V}_e$ and $\bar{V}_p$ is given in (9). In the non-relativistic limit, $h \to \rho$ takes the form

$$w_\varepsilon\big|_V (\varepsilon_V\big|_w + p) = \rho c^2. \tag{55}$$

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